Formal concepts in a material world

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Question and plan

The questions:

- how can we determine which are the formal concepts of given language?
- what it is that makes them formal.

The plan:

- explicate the notion of a formal concept by the notion of a tautology
- define tautologies as the true sentences which are not about any objects
- propose and discuss a criterion of aboutness
- loose ends

Tautologies

Our starting point:

> "Even if there *were* propositions of [the] form "M is a thing" they would be superfluous (tautologous) because what this tries to say is something which is already *seen* when you see "M"." (NDM: 111)

From tautologies to formal concepts:
Find a suitable axiomatization of your set of tautologies and the primitive vocabulary will be the formal concepts.

Examples

formal concept	*tautologies*
"$\ldots \vee \ldots$"	"$p \vee \neg p$", for any propositional constant p
"$\forall \ldots (\ldots)$"	"$\forall x(Fx \rightarrow Fx)$", for any predicate F
"$\ldots = \ldots$"	"$a = a$", for any individual constant a
"... is a number"	"n is a number", for any numeral n
"... is an object"	"a is an object", for any individual constant a
"... is a proposition"	"p expresses a proposition", for any propositional constant p
"... is a state of affairs"	"Fa is a state of affairs"
"... is complex"	"Every state of affair is complex"
"... is a part of ..."	"x is part of $x \oplus y$" (where "$\oplus$" denotes fusion), for any x and y to which the laws of mereology may be applied

Marks of tautologies

truth under all admissible substitutions:

> "The combination of symbols in a tautology cannot possibly correspond to any one particular combination of their meanings - it corresponds to every possible combination..." (NDM 118)

truth by choice of linguistic framework:

> "That M is a *thing* can't be *said*; it is nonsense: but *something* is *shewn* by the symbol "M". In [the] same way, that a *proposition* is a subject-predicate proposition can't be said: but it is *shown* by the symbol." (NDM 109, cf. TLP 4.126)

Problems:
Truth under all admissible substitutions is truth of all sentences of the same *form*, where material concepts are replaced by other material concepts.
The sentences made true by choice of a linguistic framework are those containing 'essentially' only words that give our language its structure, i.e. formal concepts.

Aboutness

The general idea:
Tautologies are those sentences that are true without being made true. They are true without being *about* any objects at all.

The first criterion

(1) p is about a $:\Longleftrightarrow$ p depends existentially on a

Problems:

- ontological dependence
- context-relativity
- necessary existents and necessary, but non-essential connections between different existents

(2)

$[p]_w$ is about $a \iff \forall v\,([p]_w$ exists in $v \rightarrow a$ exists in $v)$

material import

(3) $[p]_w$ is about a $:\Longleftrightarrow$ $\exists X$ ($[p]_w$ has material import X $\wedge$ $a \in X$)

(4) X is the material import of $[p]_w$ $:\Longleftrightarrow$ $\Box_{[p]_w}([p]_w$ exists $\rightarrow \forall a \in X(a$ exists$))$

Truth-in and truth-of

Grammar:
{Omnis, Quaedam, Nulla} | {propositio, affirmativa, negativa} | {est, non est} | {affirmativa, negativa}

Semantics:

- p is true *in* a world w iff its latin translation belongs to w (the proposition it expresses is then said to exist in w) and is true of the latin sentences in w.

- p is true *of* w iff it describes w correctly.

- p is *possibly-true* iff there is a world in which it is true.

- p is *possible* iff there is a world of which it is true.

- p is *necessarily-true* iff there is no world in which it is false.

- p is *necessary* iff it is true of all the worlds.

The criterion

General idea:
p is about X iff "truth-in" and "truth-of" coincide for it in, but only in, worlds where X exists.

The criterion:

(5) X is the material import of p in $w :\Longleftrightarrow$

$\forall v\ (\forall a \in X(\ a$ exists in $v) \rightarrow ([p]_w$ is true in $v \vee [p]_w$ is false in $v)) \wedge$

$(\forall a \in X(\ a$ does not exist in $v) \rightarrow ([p]_w$ is false of $v \wedge \neg([p]_w$ is false in $v)))$

Examples and Problems

- "All men are mortal" about all actually existing men
- "All immortals are non-men" about all actually existing immortals
- "Sokrates walks" and "Sokrates does not walk" about Socrates
- "It is not the case that Socrates walks" not about Socrates, but about everything walking
- "All men are men" and "All tall men are men" not about anything
- "There is a man" and "A man walks" about all men

Problems:

- truth-of
- necessary and impossible objects

Tautologies defined

(6) $[p]_w$ is a formal proposition $:\Longleftrightarrow$

$([p]_w$ is necessarily-true $\vee$ $[p]_w$ is necessarily-false)

(7) p is a tautology $:\Longleftrightarrow$

$[p]_@$ is true of @ $\wedge$ $\forall w([p]_w$is false of $w \rightarrow [p]_w$ is a material proposition)

Loose ends

Intuition:
Tautologies presuppose at least the existence of concepts expressed by the words they are composed of.

Good to have:
An account of truth-of doing justice to it.

Scenarios?

Analyticity

(8)
If "tail" would mean "leg", horses would have four tails.

(9)
If "tail" means "leg", horses have four tails.

A tautology p, then, is necessarily-true in the sense that "If q were true, p would be false" is false for any q. But there is more to it than this. Equally, there are no q's (in particular no propositions supposing our language to be other than it is) that make "If q is true, p is false" come out true.