What natural numbers might be

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The plan

- **HP**
- **HP^{-}**
- Frege on counting
- numerical quantifiers
- semantic relationism
- numbers are argument-places

Our starting point

(1) There is a bijection between the Fs and the Gs.

Hence, the number of Fs $\quad = \quad$ the number of Gs

HP For any concepts F and G,
there is bijection between the F and the G iff
the number of Fs = the number of Gs

HP′ $\forall F, G\ \exists R\ \forall x\ ((Fx \rightarrow \exists y(Gx \wedge$
$\forall z(R(x,z) \rightarrow z = y)) \wedge Gx \rightarrow \exists y(Fy \wedge$
$\forall z(R(z,x) \rightarrow z = y))) \longleftrightarrow\ \mathsf{N}x(Fx) = \mathsf{N}x(Gx)$

If there are numbers

$\mathbf{HP}^-$ For any concepts F and G, if there is such an object as the number of Fs or the number of Gs, there is bijection between the F and the G iff the number of Fs $=$ the number of Gs

$\mathbf{C}$ For any concept F, there is such an object as the number of Fs

Frege on counting

"Wenn wir die zu einem Begriffe $\Phi(\zeta)$ gehörende Anzahl bestimmen, oder, wie man gewöhnlich sagt, wenn wir die unter den Begriff $\Phi(\zeta)$ fallenden Gegenstände zählen, so ordnen wir diese den Zahlwörtern von Eins an der Reihe nach zu bis zu einem Zahlworte "N", das dadurch bestimmt wird, dass die zuordnende Beziehung den Begriff $\Phi(\zeta)$ in den Begriff "Glied der Reihe der Zahlwörter von "Eins" bis "N" " und dass die umgekehrte Beziehung diesen Begriff in jenen abbildet. Dann bezeichnet "N" die gesuchte Anzahl; d.h. N ist diese Anzahl." (*Grundgesetze*, §108)

"If we determine the number belonging to a concept word $\Phi(\zeta)$ – or, as one ordinarily says, if we count the objects falling under the concept $\Phi(\zeta)$ – then we successively co-ordinate these objects with the number-words from "one" up to a number-word "N". This number-word is determined through the co-ordinating relation's mapping the concept $\Phi(\zeta)$ into the concept "member of the series of number-words from "one" to "N" " and the converse relation's mapping the latter concept into the former. "N" then designates the number sought; i.e. N is this number."

The "radical adjectival strategy"

(2) $\exists x, y$ (x is needed to fix a lightbulb $\wedge$ y is needed to fix a lightbulb $\wedge$ $x \neq y$ $\wedge \forall z$(z is needed to fix a lightbulb $\rightarrow (z = x \vee z = y))))$

$$
\begin{aligned}
0 \quad &\rightsquigarrow \exists_0 x Fx &&:= \forall x \neg Fx \\
1 \quad &\rightsquigarrow \exists_1 x Fx &&:= \neg \forall x \neg Fx \wedge \forall x \forall y ((Fx \wedge Fy) \\
& && \qquad \rightarrow x = y) \\
n+1 \quad &\rightsquigarrow \exists_{n+1} x Fx &&:= \exists x (Fx \wedge \exists_n y (Fy \wedge x \neq y))
\end{aligned}
$$

Semantic relationism

Fine, “The role of variables”:

Semantic relationism:
There are external semantic relations, i.e. relations not supervenient on intrinsic semantic features of their relata.

Semantics of telegraphic notation for variables:
The simultaneous assignment of values to different variables must provide them with a coordination scheme, i.e. tell which occurrences are to be coordinated with which other occurrences of the same or different variable.

What numbers might be

Fine, “Neutral relations”

positionalism: they are argument-places

anti-positionalism: they are abstract of constituents in relational complexes with respect to the relation co-positionality