The reducibility of relations

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The plan

- Logic of relations
- Monadism
- Monism
- Converses and neutrality
- Foundations
- Factorisation
- Distributional properties of fusions

Russell's critique

needed for logic?
there is no reason to expect that elimination of polyadic predicates will leave us with an equivalent system

monadism: "Rab" analysed as "$Fa \wedge Gb$"'
how to interpret the relational properties?

monism: "Rab" analysed as "$H(ab)$"
how to structure the whole?

Monism

1. *A binary relation R is* $\wedge$-representable *iff there are monadic predicates F and G such that, for all x and y, xRy iff $Fx \wedge Gy$.*

(1) $\forall x, y, u, z((xRy \wedge uRz) \rightarrow xRz)$

$$\forall x, y, u, z, v((yRu \wedge xRu \wedge xRz) \rightarrow (yRz \vee (xRv \wedge vRu)))$$

Monadism, cntd

Leibniz:
"La raison ou la proportion entre deux L et M peut être conçue de trois façons: comme raison du plus grand L au moindre M, comme raison du moindre M au plus grand L, et enfin comme quelque chose d'abstrait des deux, c'est à dire comme la raison entre L et M, sans considerer lequel est l'anterieur ou le posterieur, le sujet ou l'objet. Et c'est ainsi que les proportions sont considerées dans la Musique. Dans la premiere consideration, L le plus grand est le sujet; dans la seconde, M le moindre est le sujet de cet accident, que les philosophes appellent relation ou rapport. Mais quel en sera le sujet dans le troisieme sens? On ne sauroit dire que tous les deux, L et M ensemble, soyent le sujet d'un tel accident, car ainsi nous aurions un Accident en deux sujets, qui auroit une jambe dans l'un, et l'autre dans l'autre, ce qui est contre la notion des accidens. Donc il faut dire, que ce rapport dans ce troisieme sens est bien hors des sujets; mais que n'étant ny substance ny accident, cela doit etre une chose purement ideale, dont la consideration ne laisse pas d'etre utile."

"The ratio or proportion between two lines L and M may be conceived [of in] three several ways; as a ratio of the greater L to the lesser M; as a ratio of the lesser M to the greater L; and lastly, as something abstracted from both, that is, as the ratio between L and M, without considering which is the antecedent, or which the consequent; which the subject, and which the object. [. . .] In the first way of considering them, L the greater is the subject, in the second M the lesser is the subject of that accident which philosophers call *relation* or *ratio*. But which of them will be the subject, in the third way of considering them? It cannot be said that both of them, L and M together, are the subject of such an accident; for if so, we should have an accident in two subjects, with one leg in one, and the other in the other; which is contrary to the notion of accidents. Therefore we must say that this relation, in this third way of considering it, is indeed *out of* the subjects; but being neither a substance, nor an accident, it must be a mere ideal thing, the consideration of which is nevertheless useful."

Monadism, cntd

Russell (1903, §214):
The supposed adjective of L [“greater than M”] involves some reference to M; but what can be meant by a reference the theory leaves unintelligible. An adjective involving a reference to M is plainly an adjective which is relative to M, and this is merely a cumbrous way of describing a relation. [. . .] Apart from M, nothing appears in the analysis of L to differentiate it from M; and yet, on the theory of relations in question, L should differ intrinsically from M. Thus we should be forced, in all cases of asymmetrical relations, to admit a specific difference between the related terms, although no analysis of either singly will reveal any relevant property which it possesses and the other lacks.

Monism

Russell (1903, §215):
"(ab) [the whole composed of a and b] is symmetrical with regard to a and b, and thus the property of the whole will be exactly the same in the case where a is greater than b as in the case where b is greater than a. [. . .] In order to distinguish a whole (ab) from a whole (ba), as we must do if we are to explain asymmetry, we shall be forced back from the whole to the parts and their relation. For (ab) and (ba) consist of precisely the same parts, and differ in no respect whatever save the sense of the relation between a and b."

Converses and neutrality

Russell (1903, §94):
"A relational proposition may be symbolized by aRb, where R is the relation and a and b are terms; and aRb will then always, provided a and b are not identical, denote a different proposition from bRa. That is to say, it is characteristic of a relation of two terms that it proceeds, so to speak, *from* one *to* the other. [. . .] It must be held as an axiom that aRb implies and is implied by a relational proposition $bR'a$, in which the relation R' proceeds from b to a, and may or may not be the same relation as R. But even when aRb implies and is implied by bRa, it must be strictly maintained that these are different propositions."

But Fine (2000, 4):
". . . it is hard to see how the state s might consist *both* of the relation *on top of* in combination with the given relata and of the relation *beneath* in combination with those relata. Surely if the state is a genuine relational complex, there must be a *single* relation that can be correctly said to figure in the complex in combination with the given relata."

Foundations

(2) R = *having* R *to a part of a* $\bar{R} \oplus$

having $\dot{R}$ *to a part of a* $\bar{R}$

Factorisation, distributional properties

$aRb \iff (\lambda x(x \text{ has } R \text{ to } b) \oplus \lambda y(y \text{ has } \dot{R} \text{ to } a))\,(a \oplus b)$

$aRb \iff \lambda x(x \text{ has } R \text{ to } b)a \wedge (\lambda x(x \text{ has } R \text{ to } b)a \leftrightarrow \lambda x(x \text{ has } \dot{R} \text{ to } a)b) \wedge (\lambda x(x \text{ has } R \text{ to } b)a \leftrightarrow (\bar{R}_a a \wedge \bar{R}_b b))$

Fisk (1972, 147):
Relational properties are not correlated simply because of a similarity of truth values of corresponding atomic sentences. They are correlated since a relatum's having one of them depends on the other relatum's having the other. [. . .] . . . the basing of relational properties on foundations is equally a matter of dependency and not of a similarity of truth values. Relational properties are properties that relata have because there are certain foundations in those relata.