Operations, functions, relations and dependence

PALMYR 7, Paris, 6 October 2008 Philipp Keller

University of Geneva
philipp.keller@lettres.unige.ch

Abstract

GET Costello and Patterson (1998); Grünbaum (1973); Husserl (1891b); Burgess (1994); Divers (1996); King (1994); Kuhn (1979); Lemmon (1958); Parsons (1973); Reinhardt (1980); Tennant (2004); Baker and Hacker (2003); Black (1954); Boorse (2002); Dummett (1955); Hochberg (1971); Humberstone (1993); Johansson (2004); Menger (1953); Simons (1983); Stelzner (1976); Svenonius (1987); van Eijck (1991) Galton (2006)

HAVE IT IN THE BOOK Hylton (1994 1997), Kron (1988), Boorse (2002)

HAVE IT AS ARTICLE McGee (1996), Ule (1990); Wansing (1990) Mayer (1990); McGinn (1980); Bealer (1989); Bigelow and Pargetter (1987); Grossmann (1976); Hochberg (1995); Linsky (1988); Menger (1979); Varzi (1993)

IMPORTANT Williamson (1998)

Beck (2000)

Varieties of dependence

When we say that how things are in one domain depends on how things are in another domain, we may mean at least three different things:

1. that there is a functional dependence ("determination") of the elements of the second by the elements of the first;
2. that there is covariation ("supervenience") between property distributions;
3. that the elements of the second (or their properties) are somehow 'given by' or 'nothing but' the elements of the first (or their properties).

Arguably the most innovative conceptual achievement of Gottlob Frege, the founder of modern logic, mathematics and philosophy, is the concept of a function. Frege was the first to explicitly pose the problem that "an expression containing x" is ambiguous between the function, according to Frege an essentially unsaturated concept, and a compound name, denoting the value of that function for the argument x.

Husserl characterises this broad class as follows:

> "...in the concept of the operation resides something of the production of an object. Some sort of activity directs itself upon the given object and produces a new object ...The main thing is: operation is a manner of conceptual transformation of the given whereby something novel originates, but something such that owing to the transformation I can also regard it as given." (Husserl 1891a: 406)

Functions, in turn, may be considered extensionally, as an abstract univocal or 'functional' correspondance between two sets or just a set of ordered pairs satisfying the condition that it contains no two pairs with the same first, but different second members, or intensionally, as a prescription, or rule or algorithm, taking an argument and yielding a value determined by the argument. It is in the first sense that functions are indeterdefinable with sets (via the operations 'being the graph of' and 'being

the characteristic function of'), while it is in the second that we say that non-computable functions are 'not really' functions.

A related distinction has been made between two different senses of "different" (cf. Beck 2000), where one (*verschieden*) is a relational adjective, the other (*anders*) is a comparison operator. In the first sense, "Luise owns a different car than Otto" is formalised as "$\exists x(\text{car}(x) \wedge \text{owns}(\text{Luise}, x) \wedge (\text{different}\ (x, max(\lambda y(\text{car}(y) \wedge\ \text{owns}(\text{Otto}, y))))))$", whereas in the second it is formalised as "anders(Luise, Otto)$(\lambda z \lambda v[\text{car}(v) \wedge (\text{owns}(z, v)])$ – in the first sense, we say of the two cars that they are different, while in the second we compare Luise and Otto with respect to the car-owning property (Beck 2000: 112).

In the case of functions, the talk about the function "yielding" a value is very misleading: functions do not yield anything, they simply characterise a range of things as being functionally determined by another range. As I will argue in the following, functions do not create nor characterise dependence relations, while operations significantly do so.

This crucial difference is responsible for two important distinctions between operations and functions: there is no incoherence in taking (some) functions to be appplicable to themselves (as they are in λ-calculus), while this it is incoherent to talk about self-applying operations; operations are iterable while functions are not.

Ontological Dependence

What functions are

1 p

2 The proposition that p is true

3 It is true that p

4 (1) iff (2) iff (3)

5 If (2) $\wedge$ (3), then (2) because (3)

Maybe the above does not presuppose Prior's analysis of (2). The argument for the claim that (2) and (3) do not mean the same thing is that the functor but not the predicate/function is iterable

Truth as a predicate vs. truth as a functor

Why operations are different from functions

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