The reducibility of relations

6th eidos meeting

Abstract

Is relationality an irreducible feature of reality? Are relations reducible to monadic properties? In this talk, I argue in favour of two things: that the two questions mentioned are different; and that the answer to both questions is in the affirmative.

The irreducibility of relations

On hardly any subject has contemporary philosophy diverged as much from philosophical tradition as with respect to relations. Aristotle held that relations are "the least of the things there are"[1] and the Stoics, Averroes, William of Ockham, Hobbes and Spinoza have denied them real existence. Leibniz famously argued in his correspondance with Clarke that relations, if they existed, would be "in two subjects, with one leg in one, and the other in the other, which is contrary to the notion of accidents"[2] and he thought that "there is no denomination so extrinsic as not to have an intrinsic one for its foundation".

Though it certainly sounds anachronistic, this reductionist attitude towards relations has some intuitive plausibility. Although contemporary property theorists usually assume that what they say about properties easily generalises to relations, relations pose problems that do not arise (at least not as sharply) in the monadic special case. It is, e.g., unclear whether and where relations can sensibly said to have a location: we cannot say, it seems, that they are wholly present in all their relata, and it does not seem to make sense of speaking of parts of one relation located in different places.[3] Relations, but not properties, can be merely partially exemplified.[4] As Armstrong (1986), Forrest (1986) and Sider (1996) have noted, an 'extensional' conception of relations as n-tuples of relata commits us to the claim that either "relation" has different meanings corresponding to different set-theoretic construals of ordered n-tuplesor else that "relation" is only partially interpreted: what do, say, binary and 17-ary relations have in common they do not share with monadic properties? Relations, finally, seem to contain a mysterious "direction" component which we do not find in their monadic cousins: relations may be "from" some relatum "to" some other and it seems possible that they differ from each other in this respect alone. A property reduction of relations, therefore, would certainly simplify our metaphysical world-picture and it has been assumed, for a long time, that such a reduction could be achieved.

Things radically changed with Russell. In his *Principles of Mathematics* (?: 221 (§212)) he criticised both monadism and monism. Monadism (defended, according to Russell, by Leibniz and Lotze) holds

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[1] Cf. *Met.* N 1088a22. Ross translates this as "the relative is least of all things a real thing or substance, and is posterior to quality and quantity" (Aristoteles 1924: 1719). Aristotle argues against relations partially on the ground that they give rise to Cambridge changes (cf. *Met.* N 1088a30-1099b1).

[2] "...en deux sujets, qui auroit une jambe dans l'un, et l'autre dans l'autre, ce qui est contre la notion des accidens." (Leibniz's fifth letter to Clark, Leibniz (1890: 401), Leibniz (1956: 71)).

[3] Armstrong (1989b: 98), hesitatingly, accepts the view that they are unlocated, without explaining how a mere difference in arity could account for such a drastic difference in ontological character to properties which are located where their exemplifications are.

[4] Armstrong (1978: 77) reduces only partially instantiated relations to relational properties, but this seems ad hoc in view of his general reduction of relational properties to relations. Moreover, populating the universe with properties like *expecting the Second Coming* does not match well with his realism.

that any truth of the form "aRb" is equivalent to some truth of the form "$Fa \wedge Gb$", while monism (represented, for Russell, by Spinoza and Bradley) replaces "aRb" by some predication of the whole consisting of both relata taken together, "$H(ab)$". Against monadism, Russell urges that relational properties cannot be interpreted except as involving relations (?: 222–223 (§214)). Against monism, he argues, that it is unable to distinguish the two directions characteristic of (binary) asymmetric relations other than by distinguishing the two parts of the whole by some other asymmetric relation (?: 224–225 (§215)).

Subsequent on his influential criticism of attempts to reduce relations to properties, many philosophers bought from Russell some highly simplified account of why 20th century logic and philosophy is superior to its predecessors. Broadly Aristotelian logic and metaphysics, it was said, were crippled by their inability to get to grip with relations. Frege then freed logic from its artificial limitation to monadic properties, discovered predicate logic and opened up the way to a more solidly ontological account of relations. Only by taking relations seriously, the story goes, can we account for phenomena like multiple generality, e.g. the difference between "$\exists x \forall y(Rxy)$" and "$\forall y \exists x(Rxy)$".

In part, this story rests on indubitable logical fact. By Church's Theorem, monadic first-order predicate logic is decidable, while dyadic first-order predicate logic is not. The philosophical relevance of this difference, however, is not easily assessed: Kripke (1962) showed that monadic modal logic is likewise undecidable, and Meyer (1968) showed the undecidability of monadic relevance logic. But even if it is granted that any full predicate logic is importantly different from the logic of its monadic fragment, this is a difference in expressive power between different kinds of predicates, a difference on the level of representations not between what they represent. It is not clear whether the difference in logical behaviour of predicates cuts any ontological ice.

It is indisputable that relations are distinguished from their relata by reason. The real question, as the Stoics realised, is whether this distinction by reason (*rationis*) translates into a distinction in reality (*realiter*) or whether it is distinction only on the basis of form (*formaliter*). This question, contrary to what is often assumed, cannot be decided on purely logical grounds: it is an overstatement to say that the irreducibility of relations is a metaphysical lesson taught us by modern logic.[5]

We may acknowledge that it is useful, and even indispensable, to quantify over relations and to distinguish syntactically the symbols expressing them from the individual terms with the referents of which they form expressions for relational properties. This is not to say, however, that we have to acknowledge relations as a separate and irreducible *ontological* category. Leibniz, after all, did not deny the reality of relations. He just held them to be "in the mind alone".[6] In the same vein, we may very well hold relational *talk* is irrreducible, while still attempting an ontological reduction of relations. The reduction to be attempted, in other words, does not aspire to express logical laws – there is therefore no reason to expect that the elimination of polyadic predicates will leave us with an equivalent system (cf. Fisk 1972: 149).

What would such an ontological reduction of relations amount to? Humberstone (1996: 219) has given the following definition of monadism:

Definition 1. *A binary relation R is $\wedge$-representable iff there are monadic predicates F and G such that, for all x and y, xRy iff $Fx \wedge Gy$.*

More generally, call a relation R $\circ$-representable for any truth-functional connective "$\circ$" if there are monadic predicates F and G such that $\forall x, y(Rxy \leftrightarrow (Fx \circ Gy)$. A relation is monadically representable iff it is $\circ$-representable for some truth-functional connective "$\circ$" (Humberstone 1984: 366). Humberstone (1984: 369-370) has shown that a binary relation R is $\wedge$-representable iff it satisfies the following condition of "forgetful transitivity" (cf. also Humberstone 1996: 219, 259) :

$$\forall x, y, u, z((xRy \wedge uRz) \rightarrow xRz) \tag{1}$$

[5] This is the view of ?: 37: "...it is one of the few unequivocal metaphysical lessons of modern logic that relations are indispensable to an account of the world. It's all very well to fantasize them as a "supervenient" free lunch; but save for ontological anorectics, the consequent inanition holds little charm, least of all in desert landscapes."

[6] Cf. his letter to des Bosses: "Orders, or relations which join two monads, are not in one monad or the other, but equally well in both at the same time, that is, really in neither, but in the mind alone."

In general, a binary relation is monadically representable iff it satisfies the following condition (Humberstone 1984: 373–375):

$$\forall x, y, u, z, v((yRu \wedge xRu \wedge xRz) \rightarrow (yRz \vee (xRv \wedge vRu))) \quad (2)$$

As this obviously is not true of all dyadic relations, not all such relations are monadically representable. We cannot, therefore, do away with truly relational (more than one-place) predicates and thus monadism, in its simplest form, is false.[7]

Could we, e.g., 'reduce' relations to relational properties, properties of the form *standing in* R *to* a for some relation R and some particular a? Hochberg (1988: 196), e.g., has argued that the answer is no: relational properties do not give us expressive power enough to state even their own identity conditions. To say that, generally and as a matter of logical truth, if $a = b$, then $\lambda x(aRx) = \lambda x(bRx)$, we need to quantify over relations. The individuation of relational properties, then, presupposes a prior individuation of relations – this part of Russell's critique also seems justified. Furthermore, a 'reduction' of relations to relational properties is in danger of being trivial: a thesis according to which the fact that aRb is 'really' the fact that $(\lambda x(xRb))a \wedge (\lambda y(aRy))b$ but that does not substantiate any sense in which the latter is in some sense prior to the first is not of much interest.

How could such a priority be spelt out? Supervenience, nowadays, is the natural candidate. Supervenience allows us to say that relational vocabulary is ineliminable, even though its applicability "rest on and is exhausted by" monadic facts (?: 100) Supervenience, however, comes in different forms.[8] Humberstone's result shows that we cannot expect a one-to-one correlation between binary and conjunctions of monadic predicates. Should we then opt for global supervenience, some claim to the effect that no two worlds can differ in relational fact without differing in what monadic properties are exemplified?[9] Or should we just say that relational facts *presuppose* the existence of at least some monadic facts?[10] Or should we just say that relations are, in some sense, 'dependent' on their terms?[11]

The difficulties of a 'reduction' of relations to relational properties can be seen in the passage ?: 13 claimed to be "of capital importance for a comprehension of Leibniz's philosophy":

> The ratio or proportion between two lines L and M may be conceived [of in] three several ways; as a ratio of the greater L to the lesser M; as a ratio of the lesser M to the greater L; and lastly, as something abstracted from both, that is, as the ratio between L and M, without considering which is the antecedent, or which the consequent; which the subject, and which the object. [...] In the first way of considering them, L the greater is the subject, in the second M the lesser is the subject of that accident which philosophers call *relation* or *ratio*. But which of them will be the subject, in the third way of considering them? It cannot be said that both of them, L and M together, are the subject of such an accident; for if so, we should have an accident in two subjects, with one leg in one, and the other in the other; which is contrary to the notion of accidents. Therefore we must say that this relation, in this third way of considering it, is indeed *out of* the subjects; but being neither

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[7] This form of irreducibility can also be traced back to the treatment of multiple generality in classic predicate logic: Grossmann (1983) points out that while the left of the following inferences is valid, the one on the right-hand side is not:

$$\frac{\forall x \exists y(Fx \wedge Gy)}{\exists y \forall x(Fx \wedge Gy)} \qquad \frac{\forall x \exists y(xRy)}{\exists y \forall x(xRy)}$$

[8] We will distinguish, contrast and evaluate different concepts of supervenience in much detail in sect. ??.

[9] It is not clear, however, that even global supervenience can do without correlations between supervening and subvening predicates. Cf. sect. ?? for details.

[10] This is the option of choice for ?: 98: "It is at least a plausible thesis that a world in which there are neither substances nor monadic qualities but there are nevertheless relations, is impossible." ?: 101 uses this against Russell and argues that he failed to distinguish supervenience from reduction: "Foundationism must not allow itself to be restricted in its search for foundations by any idea that sameness of relations requires sameness of foundation." CHECK: DOES HE HAVE AN EXEMPLIFICATION-RELATIVISED NOTION OF SUPERVENIENCE?

[11] Grossmann (1983: 197ff) argues that intentional relations are not so dependent and ?: 178, n. 2 has taken this to be a proof that they are not relations at all.

a substance, nor an accident, it must be a mere ideal thing, the consideration of which is nevertheless useful.[12]

?: 222 (§214) interprets this passage as a statement of monadism (the view that "aRb" has to be analysed as "$Fa \wedge Gb$") and urges against it that relations are prior to the relational properties they give rise to:

> The supposed adjective of L ["greater than M"] involves some reference to M; but what can be meant by a reference the theory leaves unintelligible. An adjective involving a reference to M is plainly an adjective which is relative to M, and this is merely a cumbrous way of describing a relation. [...] Apart from M, nothing appears in the analysis of L to differentiate it from M; and yet, on the theory of relations in question, L should differ intrinsically from M. Thus we should be forced, in all cases of asymmetrical relations, to admit a specific difference between the related terms, although no analysis of either singly will reveal any relevant property which it possesses and the other lacks.(**?**: 222–223 (§214))

Russell's point here is not just that the allegedly subvening relational properties are not intrinsic, but that they can only necessitate the relation if they differ in some specific, asymmetric way. This asymmetric difference, however, constitutes an additional, and unreduced, relational fact.
Russell makes a related case about the monadistic theory of quantitative relations. If "a is greater than b" is analysed as being foundated on a's being 20 hectares and b's being 15 hectares, these foundations entail the relational fact only if 20 is greater than 15. The reason this regress is vicious is not just, as **?**: 103 thinks, that Russell takes monadism to be proposing an eliminative analysis of relational ***propositions***. It is because the proposed analysis of "aRb" as "$Fa \wedge Gb$" is incomplete and in need of another conjunct, "$FR'G$", where R' is another asymmetric relation. I do not see why the regress should be tolerable just because these relations "become more and more abstract":

> ...where relations are supervenient, Russell's regress is not vicious. At each step in the regress, the asymmetric relation between the foundations will become more abstract, and will soon be repeated at each successive step. [...] Regresses of successively more abstract items, even if non-terminating, are harmless. (**?**: 104)

The supervenience argument against Russell does not, it seems, answer his critique of monadism. Even if the unreduced relations become more abstract, they are still relations, and a reductionist position restricted to non-abstract or not-too-abstract relations is without much metaphysical interest.
The supervenience reply to Russell, I think, is better applied to his critique of monism which is the following:

> (ab) [the whole composed of a and b] is symmetrical with regard to a and b, and thus the property of the whole will be exactly the same in the case where a is greater than b as in the case where b is greater than a. [...] In order to distinguish a whole (ab) from a whole (ba), as we must do if we are to explain asymmetry, we shall be forced back from the whole to the parts and their relation. For (ab) and (ba) consist of precisely the same parts, and differ in no respect whatever save the sense of the relation between a and b. (**?**: 225 (§215))

[12]This is the translation by **?**: 13 of "La raison ou la proportion entre deux L et M peut être conçue de trois façons: comme raison du plus grand L au moindre M, comme raison du moindre M au plus grand L, et enfin comme quelque chose d'abstrait des deux, c'est à dire comme la raison entre L et M, sans considerer lequel est l'anterieur ou le posterieur, le sujet ou l'objet. Et c'est ainsi que les proportions sont considerées dans la Musique. Dans la premiere consideration, L le plus grand est le sujet; dans la seconde, M le moindre est le sujet de cet accident, que les philosophes appellent relation ou rapport. Mais quel en sera le sujet dans le troisieme sens? On ne sauroit dire que tous les deux, L et M ensemble, soyent le sujet d'un tel accident, car ainsi nous aurions un Accident en deux sujets, qui auroit une jambe dans l'un, et l'autre dans l'autre, ce qui est contre la notion des accidens. Donc il faut dire, que ce rapport dans ce troisieme sens est bien hors des sujets; mais que n'étant ny substance ny accident, cela doit etre une chose purement ideale, dont la consideration ne laisse pas d'etre utile." (Leibniz 1890: 401). As Russell notes, the reality of relations, for Leibniz, consists in their being perceived by God (cf. the first draft of his letter to Bartholomeus des Bosses from the 5th of February 1912, Leibniz (1879: 438)). **?**: 14 comments on this as following: "...Leibniz is forced, in order to maintain the subject-predicate doctrine, to the Kantian theory that relations, though veritable, are the work of the mind".

Russell's worry here is not, as it was in the case of monadism, how to distinguish the relational properties, but how to distinguish the parts of the whole that are said to exemplify them other than by reference to the relation to be reduced. This, however, is an epistemological, not an ontological question. Supervenience is untouched, if we ascribe, e.g., to (ab), but not to (ba) the property ***increasing in length***. This property is structural – its exemplification presupposes a certain structure among the parts of the exemplifying thing; it can be exemplified by (ab) without being exemplified by (ba).Perhaps a thesis that relations supervene on structural properties of wholes does not deserve to be called 'reduction', but it is the most we can hope for.

Any 'reduction of relations' that may sensibly be undertaken must therefore meet the following conditions: it must reduce relations to properties of some whole containing their relata;[13] it must not attempt to reduce relational talk and assign truth-conditions to it that do not quantify over relations; it must show how relational properties can be understood without reference to and are in some sense prior to relations. It is not clear, how these desiderata might be met.

By a 'relational fact', in the following, I will mean a fact that makes some proposition true in which occurs essentially a predicate of adicity higher than one. Defined in this way, "...the existence of relational facts does not automatically entail any real existence for relations" (**?**: 97). The question whether relations exist as an ultimate ontological category or whether they can be 'reduced' to properties of complexes is a matter of how to analyse relational facts.

Relational facts are peculiar in many respects. A first problem concerns the relation between relations and relational properties. Necessarily, whenever a dyadic relation R is exemplified, say by a and b, two relational properties, ***having*** R ***to*** a and ***having*** R ***to*** b, are exemplified too, by b and a respectively. If relations and relational properties are distinct, this co-exemplification tie constitutes a necessary connection between distinct existences, something which is ***prima facie*** mysterious and has to be explained or explained away.

One strategy is to deny that relational properties are ontologically basic. They differ from real properties, it might been held, by being 'impure', i.e. by involving essential reference to individuals. The problem with this, however, is that it is far from clear that all relational properties are impure. Humberstone (1996: 212–213), e.g., distinguishes three ways to obtain relational properties from relations: quantification, reflexivization and place-fixing. As Humberstone (1996: 216) points out, these are just three of the infinitely many patterns usable.

Even if we restrict ourselves to relational properties obtained by quantification and among those of which some qualitative property is predicated, the reduction is far from obvious. Armstrong (1978: 78), e.g., proposes to reduce those relational properties to relations and properties via the following equivalence (cf. also Armstrong 1997: 92):

$$\text{a has } R \text{ to an } F \iff \exists x(x \text{ is an } F \text{ and } a \text{ has } R \text{ to } x) \qquad (3)$$

It is not clear, however, that this strategy can succeed: I can hate the murderer of my friend without hating anyone in particular; I can be smaller than a unicorn without there being a unicorn that is larger than I am; there might be an elephant that is smaller than me and still I am smaller than an elephant. These examples, probably, could be explained away. The general strategy, however, is unsatisfying, as long as we have no way to deal generally with relational properties, pure and impure.

If a reduction of relational properties to relations meets with problems, can conversely relations be taken to be properties of the fusion of their relata? What is the connection between a relation linking some terms and the relationally structural property of the fusion of their relata it gives rise to? Different sorts of relations give us different sorts of properties of the fusion of their relata. Relational properties allow us to factor out the contributions the parts make to these properties of the whole. It is therefore natural to ask whether we can not only account for relational properties in terms of relations, but also go the other way round and explain the holding of a relation by the exemplification

[13]Armstrong (1978: 71) called such properties of fusions "relationally structural properties", though he has subsequently changed his mind about these matters: Armstrong (1989a: 106) thinks that (at least) internal relations are non-relational structural properties of the fusion of their terms. External relations, on the other hand, are now no longer taken to be properties of the fusion of their relata but non-mereological constituents of a state of affairs containing them (Armstrong 1997: 104).

of relational properties. A first candidate accomplishing this is the following (letting "$\dot{R}$" denote the converse of R), where the intrinsic nature of a is taken to be the totality of a's intrinsic properties:

$$\text{a has } R \text{ to } b \iff \exists F, G \; (F \text{ is the intrinsic nature of } a \wedge G \text{ is the intrinsic nature of } b \wedge a \text{ has } R \text{ to an } F \wedge b \text{ has } \dot{R} \text{ to a } G) \quad (4)$$

This will not do, however, even apart from the reference to the converse of R we will later problematise. Define external relations to be those that do not supervene on the intrinsic properties of their relata, but on the intrinsic properties of the n-tuples of their relata (or, equivalently, of the fusion of their relata).[14] Such relations can differ between duplicates, i.e. things with the same intrinsic nature. For such external relations (4) does not hold, for then ***having*** R ***to*** a is not equivalent to ***having*** R ***to something that has*** a***'s nature***.[15] The reason for this, I think, that taking the pairs as the supervenience basis is not enough: we have to go for the fusions.

Just fusions, however, will not do neither, as Russell's argument against monism shows: they have to be structured, their parts have to be distinguished and we need to be able to talk of a specific order of their parts. We need to be able to say that (ab) differs from (ba) not by containing different parts but by being composed of these parts in some different way. How could this be achieved without talking of relations holding between these parts? This will be the question addressed in the next section. Before tackling it, however, we have to some other puzzling features of relations.

Relational facts

A first, an important question we have to address is whether relations have a definite adicity. Relations, while distinguished from properties by being expressed by predicates taking more than one argument place, often connect an indefinite, rather than fixed, number of particulars. Such are the variably polyadic relations expressed by "...cause $\cdots$", "...are adjacent", "...are consistent", "...are brothers", "...built the bridge" etc. Adam Morton (1975) proposed to analyse them using M-quantifiers which stand for infinite conjunctions of ordinary quantifiers. "Any people who live together will influence each other" then goes over into "$\forall[x](T[x] \rightarrow I[x])$", which is an abbreviation for "$\forall x(Tx \rightarrow Ix) \wedge \forall x, y(Txy \rightarrow Ixy) \wedge \forall x, y, z(Txyz \rightarrow Ixyz) \wedge \dots$" (Morton 1975: 310). Morton's semantics associates any multigrade relation with a set of tuples of different adicities. Symmetrical multigrade relations, i.e. relations that symmetrically relate two groups of an indefinite number of members, can then be interpreted in straightforward mereological terms: to say that Adam, Bill and Charles live together is to say that the whole composed of Adam, Bill and Charles lives* together, where "...lives* together" is a predicate that applies to wholes composed of people living together.[16]

Multigrade relations, then, are not so much a problem of semantics; they are naturally interpreted as monadic properties of wholes. They present, however, a metaphysical problem. How can we say, e.g., that "$I[x]$", where "$[x]$" is a 'multigrade variable', stands for the relation of living together? How can we say, as we should, that it takes at least two particulars to live together?[17]

Another difficulty pertains to reflexive relations. Is it right to classify open sentences such as "$\lambda x(Rxx)$" as constituents of relational facts? Is identity, e.g., a ***relation*** in which every thing stands to itself? To think it is not testifies, according to Humberstone (1996: 213), of a conflation of the relationality of a property with its extrinsicness: "Raa" to be sure, may ascribe an intrinsic property to

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[14]Relations that supervene on the intrinsic properties of their relata taken individually are called "internal", such that do not supervene on either kind of intrinsic properties are neither internal nor external, but extrinsic.

[15]Presumably for this reason, Armstrong (1978: 78) has argued that properties like ***having*** R ***to*** a do not count as relational properties, as they are expressed by 'impure' predicates (cf. also Armstrong 1997: 93).

[16]The case of non-symmetrical relations is slightly more complicated: The asymmetry between pluralities, however, has to be preserved: it does not follow from the fact that Adam fought with Bill and Charles that Bill fought with Adam and Charles (Morton 1975: 318, fn. 3).

[17]Note, incidentially, that Morton's rendering of "Any people who live together will influence each other" is only true if it is true if "Ix" and "Tx" are true of every individual in the domain. This seems to rule out reading them as "...live together" and "...influence each other" respectively.

a, but this is compatible with the property in question being relational.Broadly, the distinction is that the first notion applies to properties and relations and the second to predicates. Even if we concede that "Raa" may express the holding of a relation between a and itself, how should we interpret it? Does it assert the holding of the same relation that "Rab" ($a \neq b$) asserts to hold between a and b? Geach (1975) argued that there is a clear and logically important sense in which "Brutus killed Brutus" and "Cato killed Cato" contain a common predication which they do not share with e.g. "Brutus killed Caesar". Suppose, then, we distinguish between "...kills $\cdots$" and "...kills him- or herself". We would then treat "$\lambda x(Rxx)$" as expressing a monadic property. As (Hochberg 1988: 195) argued, this has its drawbacks: how would we then describe the holding of an asymmetric relation R between a and b? Not as "R holds between a and b, but not between a and itself", because the conjuncts would not assert that one and the same relation holds between one pair but not between the other. The difficulty, then, is this: find a semantic difference between the 'predicative' parts of "Brutus killed Brutus" and of "Brutus killed Caesar" without postulating two relations.

Reflexive relations are peculiar in other respects too. If we accept "$\lambda x(Rxx)$" as expressing a monadic property, it will be identical with its converse. Russell, in the *Principles of Mathematics*, argued that *in all other cases*, relations are different from their converses:

> A relational proposition may be symbolized by aRb, where R is the relation and a and b are terms; and aRb will then always, provided a and b are not identical, denote a different proposition from bRa. That is to say, it is characteristic of a relation of two terms that it proceeds, so to speak, *from* one *to* the other. [...] It must be held as an axiom that aRb implies and is implied by a relational proposition $bR'a$, in which the relation R' proceeds from b to a, and may or may not be the same relation as R. But even when aRb implies and is implied by bRa, it must be strictly maintained that these are different propositions. (**?**: 95–95 (§94))

It seems difficult to agree with Russell. If the *propositions* "aRb" and "bRa" are different, where does this difference come from? Not from a and b, so the relations must be different; they differ, Russell would say, in their *sense*. But then R and $\dot{R}$ ("R'") also differ in sense and hence cannot be the same relation, contrary to what Russell asserts.

Russell seems to think of the relational complexes aRb and $b\dot{R}a$ as consisting of *four* constituents: a, b, the relation R and its 'sense'. But what could this 'sense' possibly be? Moreover, the exclusion of the case where a and b are identical seems justifiable only if the relation denoted by "R" in "aRa" *lacks* a direction (a 'sense') – but how could the mere fact that a has the relation to itself do away with this feature of the relation?

We face a double dilemma, for both symmetric and asymmetric relations: If R and R' are different if they have different senses, then the relational fact aRb is different from the relational fact bRa even if R is necessarily symmetric. If they can be identical, even if their senses are different, then what distinguishes aRb from bRa for asymmetric relations R? If the 'sense' of a relation is something over and above the order of its constituents, then how can we identify aRb with $a\dot{R}b$ for necessarily symmetric R? If it just consists in this order, how can we distinguish $a\dot{R}b$ from aRb for asymmetric relations? It just does not seem possible to hold both $a\dot{R}b = aRb = bRa$ for symmetric relations and $a\dot{R}b \neq aRb \neq bRa$ for asymmetric relations.

The distinction between relations and the monadic properties they give rise to pushes the ontological extravagancy still further: If even necessarily symmetric relations are different from their converses, we get four analyses of "aRb" for some symmetric relation R as monadic predications (cf. **?**: 98 (§96)): a's being R-related to b, a's being $\dot{R}$-related to b, b's being R-related to a and b's being $\ldots R$-related to a. These four relational facts are all different, in virtue of containing different constituents. How are they related? Is one of them more basic than the others? Can they be reduced to one of them? We do not want to populate the universe with distinct, but necessarily connected existents; we do not want to postulate four states of affairs where, intuitively, there is just one which admits of different analyses.

This ontological worry is as old as the rehabilitation of relations. How can it be, **?**: 14, 406 asked in the spirit of Leibniz, that $(\lambda x(aRx))b$, $(\lambda y(yRb))a$ and $(\lambda x, y(xRy))(a, b)$ represent (are logical forms of) the same proposition, given that they have different components? If they represent the same proposition, and stand for the same fact, however, what are their constituents? If relations are different from their converses, what could give us a reason to take one, but not the other, to be a constituent of a relational fact?

> ...it is hard to see how the state s might consist *both* of the relation *on top of* in combination with the given relata and of the relation *beneath* in combination with those relata. Surely if the state is a genuine relational complex, there must be a *single* relation that can be correctly said to figure in the complex in combination with the given relata. (Fine 2000: 4)

As Castañeda has remarked *à propos de* Plato, a's having R to b and b's having $\dot{R}$ to a are better thought of as two *prongs* of one state of affairs.[18] If we reduce a's having R to b to a's having the property *having* R *to* b and b's having the property *having* $\dot{R}$ *to* a, we have reduced the relation to two *different* properties, which are bound together by a "law of joint exemplification" (Castañeda 1972: 470). What is the ontological ground of such a necessary connection between distinct (non-overlapping) entities? It is not just multiplication of entities that is at stake. Another problem is indeterminacy, both ontological and semantical. Armstrong (1997: 91), e.g., claims that $\dot{R}$ is not an increase in being, for every state of affairs containing it is identical with one containing just R. He does not tell us, however, which relation is a constituent of this state of affairs. Williamson (1985) asks us to imagine two languages L' and L'', both differing from our language L only by inversing the order of arguments following R and by replacing R by its converse $\dot{R}$ respectively. By hypothesis, we cannot distinguish between L' and L''. If relations were different from their converses, we could never distinguish our language from either L' or L'' – we would never be able to know what our relational expressions are standing for. In both cases, the natural reaction is to say that there is no real question because for any relation R, R and $\dot{R}$ are identical.[19] But how can they be identical, if they apply to the same relata only if these are taken to be in different orders respectively?
The notion of a converse is tightly tied up with the notion of order – a relation takes its arguments in some specified order which depends on whether we are think of it as "...stabs $\cdots$" or as "$\cdots$ is stabbed by ...": "R" does not just stand for the relation R, but for "R with a particular convention as to which flanking name corresponds to which gap in R" (Williamson 1985: 257).[20] The connection between relations and order motivates the view that only non-symmetrical relations are "truly relational" (Armstrong 1997: 91).[21] If we have relations different from their converses, we have order; if we have order, on the other hand, we can define, for any relation, the converse of that relation and this will, for non-symmetrical relations, give us a different entity.
To solve the problem of converses, we need to loosen the connection between relations and order. This is what has been undertaken by Kit Fine (2000). He argues that for some relations, the notion of a converse does not even make sense (Fine 2000: 6). Such "neutral relations", as he calls them, do not hold of their arguments in any specifiable order (Fine 2000: 3). His starting point, as Castañeda's, is the apparent absurdity of the claim that the fact of a's being to the right of b is different from the fact of b's being to the left of a. Fine's conclusion is similar to Williamson's: we cannot, in general, speak of the "first" and the "second" argument of some relation, identifying these in terms of closeness to the relational expression or their spatial position with respect to it. Instead,"argument places in different relations can be associated only in terms of the content of the relations [...] To understand 'Rxy' and

[18] "The sentence "Simmias is taller than Socrates" does not reveal the truth it expresses perspicuously, because this sentence mentions only one Form, tallness, whereas the truth or fact in question involves two Forms, tallness and shortness." (Castañeda 1972: 469)

[19] Cf. Williamson (1985: 249) and Armstrong (1978: 42). Williamson's argument presupposes that relations are individuated by the semantical roles of expressions standing for them.

[20] We might conclude from this that the nominalisation "R" is "frivolous": "When we say that the relation 'heavier than' is distinct from the relation 'lighter than' we are then, if we are consistent, exploiting frivolous nominalization. What we are in effect saying is that "a bears the relation 'heavier to' to b" is not equivalent to "a bears the relation '*lighter than*' to b..." (Fox 1987: 199)

[21] Armstrong (1997: 91) even seems to deny that non-symmetrical relations can be reduced to properties, even to properties of the fusion of their relata. His example is "before", which he takes to be non-symmetrical (neither symmetrical nor asymmetrical). As he does not give any argument, however, I do not know what to say.

'Sxy' separately one needs to know, not just which relations they stand for, but which of the latter's argument places 'x' is associated with and which 'y'" (Williamson 1985: 260). Cian Dorr (2004: 180) has recently phrased this worry as the demand for an analysis of the following sentences which makes them jointly consistent:

R_1 and R_2 hold between a and b in the same direction. (5)

R_1 and R_2 hold between a and b in the opposite direction. (6)

If we give up on the idea, as both Williamson and Fine urge, that relations relate their terms in some specific order, how can we then account for their differential applicability, i.e. the fact that the loving relation may hold between Don José and Carmen but fail to hold between Carmen and Don José? Fine presents us with two options: positionalism, which reifies argument places and includes them as constituents into relational facts, and anti-positionalism, which takes it to be a brute fact that (some) relations may, when applied to some given terms, yield more than one relational complex.

On the positionalist account, which seems to be the one Williamson would opt for,[22] the neutral amatory relation, e.g., comes with two extra entities, the argument-places Lover and Beloved, which it associates to its terms. Exemplification of the relation must then "be understood to be relative to an assignment of objects to argument-places" (Fine 2000: 11). There are two immediate problems with this view: what entities are the argument-places that figure as extra relata of the amatory relation (Fine 2000: 16)? how could there be strictly symmetrical relations, e.g. relations R such that a's being R-related to b is the *same relational fact* than b's being R-related to a (Fine 2000: 17–18)? Because the argument positions are different and assigned to different entities, the relational facts will be different, though necessarily connected.

Anti-positionalism eschews these problems. According to the anti-positionalist, "it is a fundamental fact [...] that relations are capable of giving rise to a diversity of completions in application to any given relata and there is no explanation of this diversity in terms of a difference in the way the completions are formed from the relation and its relata" (Fine 2000: 19). Differential applicability is then explained by relations being 'completed' by their relata in the same manner as in some exemplary relational fact: the amatory relation, e.g., holds between Don José and Carmen in the same way as it holds between Abelard and Eloise, but not in the same way as it fails to hold between Carmen and Don José. We account for strict symmetry and variable polyadicity by reference to the content of the relations concerned: some relations yield a unique whole when applied to some relata, and some relations combine with an indefinite number of things.

The notion of co-mannered completion, however, is problematic. Are we really speaking of Abelard and Eloise when we say that Don José loves Carmen? Fine (2000: 23) suggests that we may understand it as involving a rigid reference to some manner of completion, which is the equivalence class of all co-mannered completions of the same relation, and to which reference is fixed by some exemplary relational fact. Part of the worry remains, however: When we say, of Carmen and Don José, that the latter loves the former but not vice versa, are we really saying that the amatory relation holds of them in some manner, but not in another? Would they love each other in both manners if they each loved the other? And if they loved each other, would they love each other in the same way than Abelard loves Eloise? The answer, as Fine (2000: 24, fn. 13) notes, is no, for otherwise Eloise would love Abelard. We thus have at least three ways two persons may love each other (three ways in which the amatory relation may be completed by two persons): from the first to the second, from the second to the first and reciprocally. For an n-place relation, there will be up to $2^n - 1$ ways it may be completed by its n relata. If there is no upper bound to the adicity of asymmetrical relations and if the number of actual individuals is finite, we might run out of exemplars – and should we not still be able to say that some relation is completed by all the actual individuals in such-and-such a manner, though there are other, pairwise different manners in which it might have been completed?

Moreover, it seems that we should allow for some modal flexibility in the choice of the paradigm pairs. If Abelard and Eloise is our paradigm pair for the amatory relation, we should not be forced to say that no-one could love someone else if Abelard and Eloise did not exist or even if there were no particulars

[22] At least on Fine (2000: 1, fn.)'s interpretation of (Williamson 1985).

whatsoever.[23]

Fine (2000: 25–26) suggests that we might define co-mannered completion in terms of substitution and say that some relational fact s is a completion of R by $a_1, \ldots, a_n$ in the same manner that t is a completion of R by $b_1, \ldots, b_n$ iff s results from t by simultaneously substituting $a_1, \ldots, a_n$ for $b_1, \ldots, b_n$. This substitution must, on an ontological level, be uniform, i.e. replace all and only identicals with identicals. As Fine has argued in more recent work, this is not enough: on a semantic level, it must preserve the semantic relations between the terms $a_1, \ldots, a_n$ for $b_1, \ldots, b_n$. It is in this coordination, I will argue, that the order-component of neutral relations is most plausibly located.

Interestingly, the problem arose already for the 'positionalist' Williamson. If relations are identical to their converses, Williamson (1985: 258) asked, what prevents $\lambda x, y(Rxy \wedge Sxy)$ from being identical to $\lambda x, y(Rxy \wedge Syx)$? Williamson's answer was: only our understanding of how the variables are to be coordinated *across* the open sentences:

> Consider a sentence such as $\forall x \exists y(Rxy \& Sxy))$. To understand '$Rxy$' and '$Sxy$' separately, one needs to know, not just which relations they stand for, but which of the latter's argument places 'x' is associated with and which 'y': this resolves the "ambiguity" between $\lambda x, y(Rxy \& Sxy)$ and $\lambda x, y(Rxy \& Syx)$: one knows in which way R and S are to be put together. The lesson is that understanding a relational expression is not simply associating it with a relation, but knowing in which way it is to be associated. (Williamson 1985: 260–261)

Suppose we want to say, with Williamson, that some relation R is identical to its converse. Are we able to say this with the following?

$$Rxy \iff Ryx \tag{7}$$

(7) only identifies a relation with its converse if we understand it as affirming that the way R connects the value of x to the value of y is the way R connects the value of y, which was the value of x, to the value of x, which was the value of y (in 'telegraphic notation', this corresponds to four gaps in (7), with lines connecting the first to the fourth and the second to the third). Only so understood does (7), rather than absurdly saying that all relations are symmetric, say what Williamson wants it to say, namely that the convention regarding the order in which the arguments are to be taken is of no ontological significance.[24] In (7), in other words, the values of the first occurrence of 'x' and the second of 'y' and of the first occurrence of 'y' and the second of 'x' have to be coordinated.

In more recent work, Kit Fine developed a theory of such coordination which forms the basis of what he calls "semantic relationism".[25] Its central tenets are that there are intrinsic semantic connections which are not reducible to and do not supervene on the intrinsic semantic features of what they connect and that these connections have to be indicated explicitly in a semantic account of the language. Under what he calls the 'default' rules of coordination, "Rxy" differs from "Rxx" by containing two non-coordinated variables.[26] The order imposed by a relation on the items it relates can then be identified with the coordination it achieves among the corresponding variables, i.e. the coordination scheme which is an equivalence relation on the expressions evaluated. The neutral amatory relation,

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[23] This, at least, is Cian Dorr's intuition for the relation of temporal order: "...intuitively, it seems that some things could have been before other things even if *no* actual particulars, or anything like them, had existed." (Dorr 2004: 169–170, fn. 18)

[24] This is not quite how Williamson puts it, cf.: "$R = \dot{R}$ allows the substitution of '$\dot{R}$' for 'R' only in contexts where they are singular terms. Thus one cannot argue from '$Rxy \equiv Rxy$' to '$\dot{R}xy \equiv Rxy$' since in 'Rxy', 'R' is a relational expression. Although it stands for the relation R, this does not exhaust its semantic significance: it stands for R *with a particular convention as to which flanking name corresponds to which gap in* R." (Williamson 1985: 257) As with the proposal of Dorr (2004: 169) on behalf of Williamson, that "bears" is systematically ambigous, this rules out reading "Rab" univocally as "the relation R holds between a and b", a reading which seems too natural to be given up easily.

[25] Cf. Fine (2003) and his John Locke Lectures for 2003, a version of which are published as *Reference, Relation, and Meaning* by Blackwell.

[26] In the above statement of the identity of a relation with its converse (7), the default rules are given up: the first occurrence of 'x' is coordinated with the second occurrence of 'y', and the first occurrence of 'y' is coordinated with the second occurrence of 'x'.

we may now say, relates Carmen and Don José in such a way that Carmen is coordinated with Eloise and Don José with Abelard. The distinction between Brutus' stabbing Brutus and Brutus' stabbing Caesar and the sense in which the former has something in common with Cato's stabbing Cato the latter has not also lies in the coordination between its relata, which is absent from Brutus' stabbing Caesar.

A surprisingly similar conclusion was drawn thirty years before. Milton Fisk (1972) argued that relations could not account for what he calls "relatedness". From Bradley's regress, he argues that if relations related, then they would have to be components of the things they relate.By Leibniz's argument, they cannot be component of both, hence they do not relate. Fisk (1972: 143) proposed to switch from the relation to the relational properties that are necessarily co-exemplified and to account for the correlation of those in terms of their monadic foundations in the different relata.

The problem with this, as Dorr (2004) has argued, that even if R_1 and R_2 are neutral relations, this is not enough to guarantee the joint consistency of (5) and (6). The completions of R_1 and R_2 by a and b are either co-mannered or there are not; we may substitute a for b or we cannot. The upshot is that we should not just divorce relational expressions from the order they singly impose among their argument places, but that there is no clear sense in which we can say that some relations orders its relata in the same manner than some other relation.

Switching to the relational properties is exactly what is needed for the joint consistency of (5) and (6) which become:

$$a \text{ is } R_1\text{-related to } b \text{ and } a \text{ is } R_2\text{-related to } b. \tag{8}$$

$$a \text{ is } R_1\text{-related to } b \text{ and } b \text{ is } R_2\text{-related to } a. \tag{9}$$

It is, of course, perfectly possible that a is both R_2-related to b and b is R_2-related to a. Does this mean that both "aR_2b" and "bR_2a" designate the same relational fact? Not necessarily. For this, both relational properties would have to be coordinated, and for this, as Fisk (1972: 144) notes, they would need to have the same foundations.

Relations and structural properties

Having settled on a anti-positionalist, but 'coordinated' account of relations, let us see where this leaves us with respect to their metaphysics.

References

Aristoteles, 1924. *Metaphysics.* Oxford: Clarendon Press. Translated by W.D. Ross, reprinted in Aristoteles (1984: 1552–1728)

Aristoteles, 1984. *The Complete Works of Aristotle – The Revised Oxford Translation*, volume 2 of *Bollingen Series,LXXI.2.* Princeton, New Jersey: Princeton University Press. Translated by Jonathan Barnes

Armstrong, David M., 1978. *A Theory of Universals: Universals and Scientific Realism, Volume II.* Cambridge: Cambridge University Press

Armstrong, David M., 1986. "In Defence of Structural Universals". *Australasian Journal of Philosophy* 64: 85–88

Armstrong, David M., 1989a. *A Combinatorial Theory of Possibility.* Cambridge: Cambridge University Press

Armstrong, David M., 1989b. *Universals: An Opiniated Introduction.* Boulder, Colorado: Westview Press

Armstrong, David M., 1997. *A World of States of Affairs.* Cambridge: Cambridge University Press

Castañeda, Hector-Neri, 1972. "Plato's Phaedo Theory of Relations". *The Journal of Philosophical Logic* 1: 467–480

Dorr, Cian, 2004. "Non-symmetric Relations". pp. 155–192

Fine, Kit, 2000. "Neutral Relations". *The Philosophical Review* 109: 1–33

Fine, Kit, 2003. "The Role of Variables". *The Journal of Philosophy* 100: 605–631

Fisk, Milton, 1972. "Relatedness without Relations". *Noûs* 6: 139–151

Forrest, Peter, 1986. "Neither Magic nor Mereology: A Reply to Lewis (1986)". *Australasian Journal of Philosophy* 64: 89–91

Fox, John F., 1987. "Truthmaker". *Australasian Journal of Philosophy* 65: 188–207

Geach, Peter Thomas, 1975. "Names and Identity". In Guttenplan, Samuel D., editor, *Mind and Language – Wolfson College Lectures 1974*, pp. 139–158. Oxford: Oxford University Press

Grossmann, Reinhardt Siegbert, 1983. *The Categorical Structure of the World.* Indianapolis, Indiana: Indiana University Press

Hochberg, Herbert, 1988. "A Refutation of Moderate Nominalism". *Australasian Journal of Philosophy* 66: 188–207. Elaborated into Hochberg (2001)

Hochberg, Herbert, 2001. "A Refutation of Moderate Nominalism". In *Russell, Moore and Wittgenstein. The Revival of Realism*, number 1 in Philosophical Analysis, pp. 175–204. Egelsbach: Dr. Hänsel-Hohenhausen

Humberstone, I. Lloyd, 1984. "Monadic Representability of Certain Binary Relations". *Bulletin of the Australian Mathematical Society* 29: 365–376

Humberstone, I. Lloyd, 1996. "Intrinsic / extrinsic". *Synthèse* 108: 205–267

Kripke, Saul A., 1962. "The Undecidability of Monadic Modal Quantification Theory". *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 8: 113–116

Leibniz, Gottfried Wilhelm, 1879. *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, volume 2. Berlin: Weidmannsche Buchhandlung. Edited by C.J. Gerhardt

Leibniz, Gottfried Wilhelm, 1890. *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, volume 7. Berlin: Weidmannsche Buchhandlung. Edited by C.J. Gerhardt

Leibniz, Gottfried Wilhelm, 1956. *The Leibniz-Clarke Correspondence*. Manchester: Manchester University Press. Edited by Hubert G. Alexander

Lewis, David K., 1986. "Against Structural Universals". *Australasian Journal of Philosophy* 64: 25–46. Reprinted in **?**: 78–107

Lewis, David K., 1999. *Papers in Metaphysics and Epistemology.* Cambridge: Cambridge University Press

Meyer, Robert K., 1968. "An Undecidability Result in the Theory of Relevant Implication". *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 14: 255–262

Morton, Adam, 1975. "Complex Individuals and Multigrade Relations". *Noûs* 27: 309–318

Sider, Theodore, 1996. "Naturalness and Arbitrariness". *Philosophical Studies* 83: 283–301

Williamson, Timothy, 1985. "Converse Relations". *The Philosophical Review* 94: 249–262