

Fundamentally, there are no relations

Philipp Keller*

May 17, 2011

Are relations fundamental?

Preliminaries:

- Question does not concern (in)eliminability of relational vocabulary.
- Russell's critique of monadism accepted (Humberstone's result).
- Russell's critique of monism accepted (Hochberg's argument: no 'reduction' to relational properties).
- Question of relative priority of relationality and structure left open, however.

Relations have (at least) two essential features properties are lacking: direction and order. If aRb we may both ask whether R holds from a to b or in the other direction and whether R holds of a and b in this or the opposite order. The two questions are different, but correlated.

Thesis: nothing that is fundamental can have both a direction and hold among its particulars in a certain order.

First problem: monadic reductions and relations as ones-over-many

If relations are one-over-many, they are individuated independently of their relata. Hence there is something that the facts that aRb , aRa and $\exists aRx$ have in common. What is it?

I can hate the murderer of my friend without hating anyone in particular; I can be smaller than a unicorn without there being a unicorn that is larger than I am; there might be an elephant that is smaller than me and still I am smaller than an elephant.

Does " Raa " assert the holding of the same relation that " Rab " ($a \neq b$) asserts to hold between a and b ? Geach (1975) argued that there is a clear and logically important sense in which "Brutus killed Brutus" and "Cato killed Cato" contain a common predication which they do not share with e.g. "Brutus killed Caesar". Suppose, then, we distinguish between "...kills ..." and "...kills him- or herself". We would then treat " $\lambda x(Rxx)$ " as expressing a monadic property. As Hochberg (1988: 195) argued, this has its drawbacks: how would we then describe the holding of an asymmetric relation R between a and b ? Not as " R holds between a and b , but not between a and itself", because the conjuncts would not assert that one and the same relation holds between one pair but not between the other. The difficulty, then, is this: find a semantic difference between the 'predicative' parts of "Brutus killed Brutus" and of "Brutus killed Caesar" without postulating two relations.

*Département de Philosophie, Université de Genève, Switzerland, philipp.keller@unige.ch.

How can it be, Ramsey (1925: 14, 406) asked in the spirit of Leibniz, that $(\lambda x(aRx))b$, $(\lambda y(yRb))a$ and $(\lambda x, y(xRy))(a, b)$ represent (are logical forms of) the same proposition, given that they have different components? If they represent the same proposition, and stand for the same fact, however, what are their constituents?

Second problem: order and direction cancel each other out

We may choose, without loss of generality, a binary relation $R(x, y)$ as our example. Suppose it holds between a and b , in this order. It follows:

- that the relational fact $R(a, b)$ is *ordered*; it has an internal structure and consists of (at least) two parts, a and b , distinguished by *how* they stand in the relation R : a is R -ing b while b is R -ed by a ;
- that, within the relation fact $R(a, b)$, R not simply holds, but holds *in a certain direction*: it holds from a to b , and is thereby different from its converse which would hold from b to a .

Armstrong and Fine argue for the identity of a 's being on top of b and b 's being below of a , and of Don José's loving Carmen and Carmen's being loved by Don José on ontological grounds.¹ But even such neutral relations, if they are relations at all, do order their relata and apply to them in a given direction. In every relational fact in which the on-top-of (i.e. the below-of) and the loving (i.e. the being loved) relations are exemplified, there is something on top and something below, someone loving and someone being loved. The following diagram

$$\begin{array}{ccc} a \rightarrow b & \xrightarrow{D} & a \leftarrow b \\ O \downarrow & & \downarrow O \\ b \rightarrow a & \xrightarrow{D} & b \leftarrow a \end{array}$$

commutes, i.e. $O(D(a \rightarrow b)) = D(O(a \rightarrow b))$. We also have $a \rightarrow b = D(D(a \rightarrow b)) = O(O(a \rightarrow b))$. The question is how this is possible, on the assumption that there is something to which the operations D and O may be applied (i.e. a relation).

Let us suppose that Othello loves Desdemona, but that Desdemona does not love Othello. Let us distinguish:

Rab Othello's loving Desdemona.

Rba Desdemona's loving Othello.

$\dot{R}ab$ Othello's being loved by Desdemona.

$\dot{R}ba$ Desdemona's being loved by Othello.

The ontology of the situation imposes three constraints that cannot be simultaneously fulfilled:

$Rab = \dot{R}ba$ There is nothing else to Desdemona's being loved by Othello than what there is to Othello's loving Desdemona. We plainly have two descriptions of the same fact.

¹Armstrong (2004: 149) calls the view that " aRb " and " bRa ", for some symmetric relation R , represent two different necessarily coexistent states of affairs a "quite serious case of metaphysical double vision".

$Rab \neq Rba$ The two facts are different because one can obtain without the other.

$Rba \neq \dot{R}ba$ The two facts are different because one can obtain without the other.

This shows that nothing can be such that both order and direction are essential to it. Order forces us to distinguish Rab and Rba . Direction forces us to distinguish Rba and $\dot{R}ba$. But their interplay forces us to identify Rab and $\dot{R}ba$. After all, converses are *defined* by

(i) \dot{R} is the converse of R : $\iff \forall x, y (\dot{R}xy \leftrightarrow Ryx)$

If we do not identify Rab and $\dot{R}ba$, we either don't mean order by "order" or we don't mean direction by "direction".

The formulation of the problem in terms of identities and differences of relational facts makes it clear that the problem arises both for theories of universals and of tropes. For universals, the question may be put as follows: both their direction and the order in which they take their arguments seem essential to relation universals, but they cannot both be. For tropes, the question becomes: how can the relational tropes in Rab and in $\dot{R}ba$ be exactly similar / of the same type / instances of the same universal, if they differ both in order and direction, and difference in order accounts for the dissimilarity (difference of type etc.) between the tropes in Rab and in Rba and difference in direction accounts for the dissimilarity between the tropes in Rba and in $\dot{R}ba$.

Relations – entities are essentially such that they apply to their particulars in a certain order and with a certain sense – cannot exist, because nothing can equally fundamentally both have a direction and order that to which it applies.

Directedness or order?

We can have order without directedness, but not directedness without order. If directed properties apply to complex objects, they thereby order them. A possible exception are vectors, which will not be discussed.²

Given that order and direction cannot be equally fundamental, we have to choose: I say that order is the more fundamental characteristic, and that the appearance of direction is the result of seeing order from a certain perspective. The directionality of relational facts is an extrinsic, description- and mind-dependent feature.

Structure without relatedness

Armstrong (1997: 35) identifies the state of affairs of a particular exemplifying a structural property with the fusion of the states of affairs of its parts exemplifying the relevant component properties. The problem with that line, however, is that the complex state of affairs, supervening on its conjuncts, is an ontological free lunch, *even though it contains as one of its (non-mereological) parts a*

²A conception of relations as vectors seems to have been the prevailing one among medieval realists and in the 16th and 17th century generally. Cf. e.g. the characterisation by Caspar Bartholin: "Five things are needed to have a relation belonging to the predicaments: [...] The Term, which is said to be that where the relation ends and rests. Thus, the son is the term of fatherhood and the father of sonhood. Terms are correlatives [...] The one is called *ad quem*, the other *a quo*, or better in quo. A relation as, for instance, fatherhood, is in a term (the father) insofar as this latter is the subject of the inherence [...] A single accident, indeed, is only in one subject, not in several ones [...]" (cited after and in a translation by Massimo Mugnai (2011).

complex universal which is not the mereological fusion of universals contained in one of its parts. This seems truly a generation of something out of nothing. The complex universal is not only not a fusion of its components, but it also contains other parts:

It may be necessary, in order to give the correct structure of a particular complex universal, to specify whether the particulars involved are identical, wholly distinct *or standing in part/whole or overlap relations.* (Armstrong 1997: 37)³

Nothing qualitative can contain such parts.

Instead of structural universals, we want *structuring* universals, universals as a result of the exemplification of which particulars become heterogeneous.

Relations supervene on (intrinsic and) extrinsic properties of their relata. Not all extrinsic properties, however, are relational: the property of the world of not containing unicorns, for example, is extrinsic but not relational.

We independently need structuring universals: It is at least possible that there are structured simples. If it is possible that there are structured simples, structure is not always the result of relatedness. If structure does not always stem from relatedness, it never does: structure is prior.

References

- Armstrong, David M., 1997. *A World of States of Affairs*. Cambridge: Cambridge University Press
- Armstrong, David M., 2004. "How Do Particulars Stand to Universals?" In Zimmerman, Dean W., editor, *Oxford Studies in Metaphysics*, volume I, pp. 139–154. Oxford: Clarendon Press
- Geach, Peter Thomas, 1975. "Names and Identity". In Guttenplan, Samuel D., editor, *Mind and Language – Wolfson College Lectures 1974*, pp. 139–158. Oxford: Oxford University Press
- Hochberg, Herbert, 1988. "A Refutation of Moderate Nominalism". *Australasian Journal of Philosophy* 66: 188–207. Elaborated into Hochberg (2001)
- Hochberg, Herbert, 2001. "A Refutation of Moderate Nominalism". In *Russell, Moore and Wittgenstein. The Revival of Realism*, number 1 in *Philosophical Analysis*, pp. 175–204. Egelsbach: Dr. Hänsel-Hohenhausen
- Mugnai, Massimo, 2011. "Leibniz's Ontology of Relations: A Last Word?" Unpublished manuscript
- Ramsey, Frank Plumpton, 1925. "Universals". *Mind* 34: 338–384. Reprinted in Ramsey (1931: 112–137); cited after reprint in Ramsey (1990)
- Ramsey, Frank Plumpton, 1931. *The Foundations of Mathematics: Collected Papers of Frank P. Ramsey*. London: Routledge and Kegan Paul, Ltd. Edited by Braithwaite, R.B.
- Ramsey, Frank Plumpton, 1990. *Philosophical Papers*. Cambridge: Cambridge University Press. Edited D.H. Mellor
- Williamson, Timothy, 1985. "Converse Relations". *The Philosophical Review* 94: 249–262

³Compare: "Consider a sentence such as $\forall x\exists y(Rxy\&Sxy)$. To understand ' Rxy ' and ' Sxy ' separately, one needs to know, not just which relations they stand for, but which of the latter's argument places ' x ' is associated with and which ' y ': this resolves the "ambiguity" between $\lambda x, y(Rxy\&Sxy)$ and $\lambda x, y(Rxy\&Syx)$: one knows in which way R and S are to be put together. The lesson is that understanding a relational expression is not simply associating it with a relation, but knowing in which way it is to be associated." (Williamson 1985: 260–261)