The Ontology of Relations

Philipp Blum

John Heil workshop
Lausanne, May 1, 2013

The Problem of Converses

The problem of converses arises because relations have 'senses':

> "A relational proposition may be symbolized by aRb, where R is the relation and a and b are terms; and aRb will then always, provided a and b are not identical, denote a different proposition from bRa. That is to say, it is characteristic of a relation of two terms that it proceeds, so to speak, *from* one *to* the other. [...] It must be held as an axiom that aRb implies and is implied by a relational proposition $bR'a$, in which the relation R' proceeds from b to a, and may or may not be the same relation as R. But even when aRb implies and is implied by bRa, it must be strictly maintained that these are different propositions." (Russell 1903: 95–95, §94)

Problem 1 : ontological profligacy.

> "If a book is on a table, *on the table* is a relational character truly predicable of the book. But this is inseparable from another relational character predicable not of the book but of the table. How are these two relational characters connected with each other? We may be tempted to say that the difference between them is purely verbal, so that, whether we say the *book is on the table* or the *table is under the book*, we are merely expressing the same fact in different language. But this cannot be true; for *being on* is different from *being under*; killing from being killed; loving from being loved. Yet it is plain that a single indivisible fact is referred to whether we say that the book is on the table or that the table is under the book." (Stout 1940: 121)
>
> "...it is hard to see how the state s might consist *both* of the relation *on top of* in combination with the given relata and of the relation *beneath* in combination with those relata. Surely if the state is a genuine relational complex, there must be a *single* relation that can be correctly said to figure in the complex in combination with the given relata." (Fine 2000: 4)

Problem 2 : indeterminacy. How can it be, Ramsey (1925: 14, 406) asked in the spirit of Leibniz's quote above, that $(\lambda x(aRx))b$, $(\lambda y(yRb))a$ and $(\lambda x, y(xRy))(a, b)$ represent (are logical forms of) the same proposition, given that they have different components? If they represent the same proposition, and stand for the same fact, however, what are their constituents? If relations are different from their converses, what could give us a reason to take one, but not the other, to be a constituent of a relational fact?

Problem 3 : regress.

> "...when we analyze them, *greater* obviously differs from *less*; thus the two propositions ["A is greater than B" and "B is less than A"] seem to be composed of different constituents, and therefore to be necessarily distinct. To deny that they are distinct, it would be necessary to hold that both *greater* and *less* enter into each proposition, which seems obviously false, or else to hold that what really occurs is neither of the two, but that third abstract relation of which Leibniz speaks [...]. In this case, the difference between *greater* and *less* would be one involving reference to the terms A and B. But this view cannot be maintained without circularity: for neither the greater nor the less is inherently the antecedent, and we can only say that, when the greater is the antecedent, the relation is *greater*, when the less, *less*." (Russell 1901: 41/300)

Fundamentally, there are no relations

I will call "relation" any property (entirely qualitative entity) that has at least one possible exemplification which involves more than one particular. Let us focus, for simplicity, on the dyadic case: we may take "R" to express a relation because it is possible that there are two things, a and b, such that aRb. That R is thusly exemplified by a and b means:

- that it imposes a direction on them: in the relational fact aRb, R goes *from* a *to* b, in so far as a is its *first* relatum and b is its *second* relatum;
- that it orders or structures a and b: in the relational fact aRb, a is R-related to b while b is such that a is R-related to it,[1] in so far as a is in the domain, while b is in the codomain of the relation R.

While properties may have directions and impose order or structure, only relations exhibit both features.

Consider the following diagram where the direction of the arrow corresponds to the direction of the relation and the order "a" and "b" to the order between a and b:

$$\begin{array}{ccc} a \to b & \xrightarrow{D} & a \leftarrow b \\ {\scriptstyle O}\downarrow & & \downarrow{\scriptstyle O} \\ b \to a & \xrightarrow{D} & b \leftarrow a \end{array}$$

This diagram commutes, i.e. $O(D(a \to b)) = D(O(a \to b))$. We also have $a \to b = D(D(a \to b)) = O(O(a \to b))$. The question is how this is possible, on the assumption that there is something to which the operations D and O may be applied (i.e. a relation). If $D = \breve{O}$, then we have an explanation, if not, we have a brute necessity. I submit that this is not possible, and one of our relations is the converse of the other. This conclusion follows without inference to the best explanation and by logic alone if we have $a \to b = O(D(a \to b))$.

Let us suppose that Othello loves Desdemona, but that Desdemona does not love Othello. We may thus distinguish four relational facts, leaving open whether they are all different:

Rab Othello's loving Desdemona.

Rba Desdemona's loving Othello.

$\breve{R}ab$ Othello's being loved by Desdemona.

$\breve{R}ba$ Desdemona's being loved by Othello.

The ontology of the situation imposes three constraints that cannot be simultaneously fulfilled:

$Rab = \breve{R}ba$ There is nothing else to Desdemona's being loved by Othello than what there is to Othello's loving Desdemona. We plainly have two descriptions of the same fact.

$Rab \neq Rba$ The two facts are different because one can obtain without the other.

$Rba \neq \breve{R}ba$ The two facts are different because one can obtain without the other.

This shows that nothing can be such that both order and direction are essential to it. Order forces us to distinguish Rab and Rba. Direction forces us to distinguish Rba and $\breve{R}ba$. But their interplay forces us to identify Rab and $\breve{R}ba$. If we do not identify Rab and $\breve{R}ba$, we either don't mean order by "order" or we don't mean direction by "direction".

Relations – entities are essentially such that they apply to their particulars in a certain order and with a certain sense – cannot exist at the fundamental level, because nothing can equally fundamentally both have a direction and order that to which it applies.

[1] Using converses, we can attribute the same property to b by "b is $\breve{R}$-related to a".

Relations cannot individuate

That relations cannot individuate was *common ground* between Russell/Moore on the one hand, and Bradley, Alexander, Bosanquet etc. on the other hand. The question is thus different from:

the axiom of internal relations whether all relations are internal in some sense of "internal"
whether relata may be bare or need to have some other properties
whether relata need intrinsic properties or whether their individuality must be intrinsic
whether individuation requires haecceities or can be done by properties at all

But then what *is* it about?

- "[I]t is impossible that the ordinals should be, as Dedekind suggests, nothing but the terms of such relations as constitute a progression. If they are to be anything at all, they must be intrinsically something; they must differ from other entities as points from instants, or colours from sounds. What Dedekind intended to indicate was probably a definition by means of a the principle of abstraction...But a definition so made always indicates some class of entities having ...a genuine nature of their own." (Russell 1903: 249)
- "If any one asserts or implies that a difference bewteen this and that can be established by the fact that this is related to one thing whereas that is related to something different, he cannot without contradiction deny numerical difference. For this and that cannot have different relations, unless the relation possessed by the one is not possessed by the other. Unless, therefore, the one has a difference from the other over and above the difference of relations, it will be true of one and the same thing that it both has and has not a given relation to something else." (Moore 1900-1901: 110)
- "The thirty-six possibilities, the one that is actual included, are (abstract) *states* of the [two] dice, not complex physical entities. Nor should any school pupil receive high marks for the question 'How do we know, in the state where die A is six and die B is five, whether it is die A or die B which is six? Don't we need a "criterion of transstate identtiy" to identify the die with a six – not the die with a five – with our die A?' The answer is, of course, that the state (die A, 6; die B, 5) is *given* as such (and distinguished from the state (die B, 6; die A, 5)). [...] The 'possibilities' simply are not given purely qualitatively (as in: one die, 6, the other, 5). If they had been, there would have been just twenty-two distinct possibilities, not thirty-six. [...] Nor, when we regard such qualitatively identical states as (A,6; B, 5) and (A, 5; B, 6) as distinct, need we suppose that A and B are qualitatively distinguishable in some other respect, say, color. [...] Finally, in setting up this innocent little exercise regarding the fall of the dice, with possibilities that are not described purely qualitatively, we make no obscure metaphysical commitment to dice as 'bare particulars', whatever that might mean." (Kripke 1980: 17–18)

References

Fine, Kit, 2000. Neutral Relations. *The Philosophical Review* 109(1): 1–33.

Kripke, Saul A., 1980. *Naming and Necessity*. Oxford: Basil Blackwell Publishers.

Moore, George Edward, 1900-1901. Identity. *Proceedings of the Aristotelian Society* 1(1): 103–127.

Ramsey, Frank Plumpton, 1925. Universals. *Mind* 34(136): 401–417. Reprinted in Ramsey (1931: 112–137); cited after reprint in Ramsey (1990).

Ramsey, Frank Plumpton, 1931. *The Foundations of Mathematics: Collected Papers of Frank P. Ramsey*. London: Routledge and Kegan Paul, Ltd. Edited by Braithwaite, R.B.

Ramsey, Frank Plumpton, 1990. *Philosophical Papers*. Cambridge: Cambridge University Press. Edited D.H. Mellor.

Russell, Bertrand Arthur William, 1901. On the Notion of Order. *Mind* 10(37): 30–51.

Russell, Bertrand Arthur William, 1903. *The Principles of Mathematics*. Cambridge: Cambridge University Press. 2nd edition: Russell (1937).

Russell, Bertrand Arthur William, 1937. *The Principles of Mathematics*. 2 edition. London: George Allen & Unwin.

Stout, George F., 1940. Things, Predicates and Relations. *Australasian Journal of Psychology and Philosophy* 18(2): 117–130.