

# Differential applicability

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## The problem

What conception of  $R$  allows us to say that

- = “ $Rab$ ” and “ $\check{R}(ba)$ ” denote the same relational fact;
- pos  $R$  holds, but  $\check{R}$  does not hold, between  $a$  and  $b$  (in this order);
- neg  $\check{R}$  holds, but  $R$  does not hold, between  $a$  and  $b$  (in this order).

To solve the problem of converses – to explain differential applicability (**pos** and **neg**) without intrinsic directions ( $\equiv$ ) –, we need to loosen the connection between direction and order. This is what has been undertaken by Kit Fine (2000). He argues that for some relations, the notion of a converse does not even make sense (Fine 2000: 7).

I find it difficult to understand this claim. Suppose we are introducing the concept of an Aristotelian universal to a class of students. We explain the general idea, and then illustrate it by pointing to a white wall. If our explanation is successful, the students will latch onto the universal whiteness, and apply “white” or “whiteness” to the qualitative characteristic the wall shares with the white sheet in front of them. Suppose we want to do the same thing by pointing to an amatory couple. Some students will latch onto  $x$ , and go on to apply “ $x$  loves  $y$ ” or “loving” to Othello and Desdemona, in this order; others, however, will latch onto  $y$  (leaving it open whether they are the same or not) and go on to apply “ $x$  is loved by  $y$ ” or “being loved” to Desdemona and Othello, in this order. Even if, as is granted,  $x$  and  $y$  are one and the same thing, the *concepts* the students acquire as a result of our (successful) introduction of the universal  $x = y$  seem clearly different, as is shown by the fact that they apply them differently to Othello and Desdemona.

We may, and must, if the argument from the problem of converses is sound, hold that both “loving” and “being loved” stand for the same relation, even though they apply to their relata in a different order. To do this, we must loosen the connection between relations and direction: even though the ‘directions’ of  $R$  and  $\check{R}$  are different, this does not distinguish them as relations. The relation, equally well denoted by “ $R$ ” or “ $\check{R}$ ”, is not intrinsically, but only extrinsically directed. In this sense, it is “undirected”, or “neutral” (with respect to direction).

“Neutral relations”, Fine (2000: 3) says, do not hold of their arguments in any specifiable order. Fine’s starting point, as Castañeda’s, is the apparent absurdity of the claim that the fact of  $a$ ’s being to the right of  $b$  is different from the fact of  $b$ ’s being to the left of  $a$ . Fine’s conclusion is similar to Williamson’s: we cannot, in general, speak of the “first” and the “second” argument of some relation, identifying these in terms of closeness to the relational expression or their spatial position with respect to it. Similarly, Fine (2000: 6) concludes that “[neutral] relations should [...] be taken to apply to their objects without regard to the order in which they might be given”.

If we give up on the idea, as both Williamson and Fine urge, that relations relate their terms in some specific order, how can we then account for their differential applicability, i.e. the fact that the loving relation may hold between Don José and Carmen but fail to hold between Carmen and Don José? Fine presents us with two options: positionalism, which reifies argument places and includes them as constituents into relational facts, and anti-positionalism, which takes it to be a brute fact that (some) relations may, when applied to some given terms, yield more than one relational complex.

Fine (2000: 9) construes both these positions as introducing further relata, “for if there were not [any further relata] and if the notion [of exemplification] were indeed order-insensitive, then we would be left with something like the attenuated form of exemplification described above and there would be no way to account for differential application”. This does not follow, however. Positionalism, while committed to argument-places or ‘positions’, can incorporate them into the relation: they can contribute to the ontology of the relational fact not by being further relata related by the relation, but rather by being essential parts of the relation itself. Similarly, anti-positionalism can hold that the similarity between co-mannered completions (relational facts in which, as a directionalist would say, the relation applies to its arguments in the same order) is extrinsic, but non-relational.

Even amended in this way, Fine’s menu of options, however, is not exhaustive: it does not follow from the fact that relations do not exhibit *intrinsic* directionality that they are not directed at all. One and the same relation,  $R = \check{R}$ , could be extrinsically directed one way when exemplified by Othello and Desdemona in this order and be extrinsically directed another way when exemplified by Desdemona and Othello in this order.

## Against directionalism

Directionalism<sup>1</sup> is the view that a relation and its converse differ in sense or direction, is defended by Russell in the *Principles of Mathematics* §219.

1. The term is from Orilia (2008: 170). Fine (2000: 1) calls it the “standard view”.

The first, quite obvious, problem of directionalism is to explain what directions are. This is important, because “without fully understanding this, it is not clear why non-symmetric and asymmetric relations should have them, whereas symmetric ones should not” (Orilia 2008: 172). Discussing his principle that the same ultimate constituents constitute different states of affairs only if they are differently organised, Armstrong (1997: 121–122) claims that (at least asymmetric) relations have a “direction”, which he represents by indexing the blanks in the corresponding state-of-affairs type. It is not clear to me, however, what this ‘direction’ is supposed to be. It cannot be a second-degree property of the relation, for we would still not be able to explain the difference between *Rab* and *Rba* and at the same time maintain that they ‘contain’ the same relation. Rather, it has to be an additional, negative, state of affairs (cf. Armstrong 2006: 193), or an ontological feature of the relation itself (an idea discussed by Armstrong (2004: 151) that Fine (2000) calls ‘positionalism’). None of the two alternatives *explains* why symmetric relations do not have directions; more importantly, neither do they explain why and how the linkage of blanks, the presence of an additional state of affairs or the exemplification of a second-order relation *makes* the relation directed.

A possible way out, surprisingly not yet defended in the literature, is to appeal to a Neo-Fregean notion of ‘content recarving’. At first sight, this could allow the directionalist to have its cake and eat it too, ie. to allow for the existence of converses and *still* to deny that the truthmakers of “*aRb*” and of “*bRa*” involve (‘substantially’) different relations, playing similar rôles in different “reconceptualisations” of the same state of affairs.<sup>2</sup> The problem with such a proposal, of course, is that it is a relabelling, not a solution of the problem: “The neo-logicist is not entitled to stipulate that there is ‘no gap’ between the obtaining of an equivalence relation and the instantiation of a novel concept. How could it ever be *stipulated* that if there is one kind of object then there is another distinct but necessarily correlated kind?” (MacBride 2003: 126). Neo-Fregean commitment to numbers must be thought of as the result of a definitional expansion of the domain, to which competent language users are entitled by their prior ontological commitments.<sup>3</sup> It is these prior ontological commitments that commit the Neo-Fregean to the view that the structure of the state of affairs in question uncovered by the abstraction principle is imposed by language. Rather than a way out for the directionalist, the Neo-Fregean approach thus concedes the non-fundamentality of relations.

## Against positionalism

On the positionalist account, which seems to be the one Williamson would opt for,<sup>4</sup> the neutral amatory relation, e.g., comes with two extra entities, the argument-places *LOVER* and *BELOVED*, which it associates to its terms. Exemplification of the relation must then “be understood to be relative to an assignment of objects to argument-places” (Fine 2000: 11). There are two immediate problems with this view: what entities are the argument-places that figure as extra relata of the amatory relation (Fine 2000: 16)? how could there be so-called ‘strictly symmetrical’ relations, e.g. relations *R* such that *a*’s being *R*-related to *b* is the *same relational fact* than *b*’s being *R*-related to *a* (Fine 2000: 17–18)? Because the argument positions are different and assigned to different entities, the relational facts will be different, though necessarily connected.

To solve both problems, we have to conceive of argument places as being part of the inner structure of relations – on the model of ‘holes’ and ‘eyes’ of trucks (Russell 1984: 86) or Fregean ‘holes’ of different shapes (Fine 2000: 10). Similarly, Williamson (1985: 260) holds that “argument places in different relations can be associated only in terms of the content of the relations [...] To understand ‘*Rxy*’ and ‘*Sxy*’ separately one needs to know, not just which relations they stand for, but which of the latter’s argument places ‘*x*’ is associated with and which ‘*y*’”. A strictly symmetrical binary relation will then, by this very fact, have just one argument place, to be filled jointly by the two things that exemplify it.

For only partially symmetric relations we need a more complicated story.

Even if relations may be considered to be intrinsically structured in a way that their nature (or ‘content’) not only determines the number and natures of its argument places, but also their ‘size’ (i.e. the range of

The positionalist solution comes at a high price: because they are part of the inner of a relation, not just the nature, but also the number of argument places is given by the relation. On this positionalist view, relations, by their very nature, have ‘fixed adicity’. But they do not.

To avoid definite adicity, we could assign roles not to relations, but to the relevant parts of relational complexes

Orilia now recognises multiple adicity and argument deletion as a problem – proposed solution: the relation has no o-roles, but is associated to o-roles “to the extent that *R* is exemplified so as to give rise to a state of affairs”: o-roles are different ways in which a relation is exemplified, ie are properties; the exemplification of eating in the relational fact of *x* eating *y* with *z*, for example, is a large fact that has 4 facts as parts – eating being attributed, *x* being the agent, *y* being the patient and *z* being the instrument.

## Against anti-positionalism

Anti-positionalism eschews these problems. According to the anti-positionalist, “it is a fundamental fact [...] that relations are capable of giving rise to a diversity of completions in application to any given relata and there is no explanation of this diversity in terms of a difference in the way the completions are formed from the relation and its relata” (Fine 2000: 19). Differential applicability is then explained by relations being ‘completed’ by their relata in the same manner as in some exemplary relational fact: the amatory relation,

2. Cf. Hale 1997: 257, 2001: 342, for discussion, cf. Potter & Smiley (2001, 2002).

3. Hale (2007: 376) and Hale & Wright (2009: 192) explicitly say so when they refuse the invitations by Sider (2007: 207) to accept quantifier variance and by Hawley (2007: 236) to accept ontological maximalism. They are certainly right in doing so: what good would it serve the Neo-Fregean if the ontological status of natural numbers were the same as of Hirsch’s incars (as Eklund (2006: 102) helpfully suggests)! Politely refusing these invitations does not, of course, commit them to Sider’s claim that ontological disputes are best held in *his* (ie. Sider’s) own language, nor – of course, cf. Hale & Wright (2009: 186, fn. 19) – does it commit them to Ross Cameron’s startling claim that “*a* exists” does not commit you to the existence of *a* (2008: 5)!

4. At least on Fine (2000: 1, fn.)’s and MacBride (2016: [30])’s interpretation of (Williamson 1985).

e.g., holds between Don José and Carmen in the same way as it holds between Abelard and Eloise, but not in the same way as it fails to hold between Carmen and Don José. We account for strict symmetry and variable polyadicity by reference to the content of the relations concerned: some relations yield a unique whole when applied to some relata, and some relations combine with an indefinite number of things.

In more recent work, Kit Fine developed a theory of such coordination which forms the basis of what he calls “semantic relationism” (cf. 2003 and 2007). Its central tenets are that there are intrinsic semantic connections which are not reducible to and do not supervene on the intrinsic semantic features of what they connect and that these connections have to be indicated explicitly in a semantic account of the language. Under what he calls the ‘default’ rules of coordination, “ $Rxy$ ” differs from “ $Rxx$ ” by containing two non-coordinated variables.<sup>5</sup> The order imposed by a relation on the items it relates can then be identified with the coordination it achieves among the corresponding variables, i.e. the coordination scheme which is an equivalence relation on the expressions evaluated. The neutral amatory relation, we may now say, relates Carmen and Don José in such a way that Carmen is coordinated with Eloise and Don José with Abelard. The distinction between Brutus’ stabbing Brutus and Brutus’ stabbing Caesar and the sense in which the former has something in common with Cato’s stabbing Cato, but not the latter, lies in the coordination between its relata, which is absent from Brutus’ stabbing Caesar.

Problems are compounded with relations of higher adicity. With respect to “ $x$  perceives  $y$ ’s hitting  $z$ ”, we need an account that simultaneously fulfills the following desiderata:

1. it should explain why the completions of the relation by  $\langle a, b, c \rangle$  and by  $\langle a, X, c \rangle$ , where  $X$  is a plural variable, may be similar;
2. it should explain why  $\langle a, a, b \rangle$  has more in common with  $\langle d, d, e \rangle$ , than with  $\langle g, h, i \rangle$ ;
3. it should identify the relational facts described by “ $x$  perceives  $y$ ’s hitting  $z$ ” and by “ $x$  perceives  $z$ ’s being hit by  $y$ ” respectively.

The neutralist has another problem, clearly stated by Russell already in 1901:

...when we analyze them, *greater* obviously differs from *less*; thus the two propositions [“ $A$  is greater than  $B$ ” and “ $B$  is less than  $A$ ”] seem to be composed of different constituents, and therefore to be necessarily distinct. To deny that they are distinct, it would be necessary to hold that both *greater* and *less* enter into each proposition, which seems obviously false, or else to hold that what really occurs is neither of the two, but that third abstract relation of which Leibniz speaks [...]. In this case, the difference between *greater* and *less* would be one involving reference to the terms  $A$  and  $B$ . But this view cannot be maintained without circularity: for neither the greater nor the less is inherently the antecedent, and we can only say that, when the greater is the antecedent, the relation is *greater*, when the less, *less*.” (1901: 41/300)

Russell’s point highlights a general weakness of all forms of resemblance nominalisms: their failure to provide truthmakers. The resemblance nominalist may, at best, hope to offer us a translation of a true description of the world, ‘*when all the facts are in*’, into a somewhat less committing idiom; he will not be able to tell us *what it takes* for a relational complex to come into being. Once we know that Othello’s love for Desdemona is unrequited, we may state the ensuing resemblances, but no account is provided of the ontological ground of this unilateral loving.

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5. In the above statement of the identity of a relation with its converse (??), the default rules are given up: the first occurrence of ‘ $x$ ’ is coordinated with the second occurrence of ‘ $y$ ’, and the first occurrence of ‘ $y$ ’ is coordinated with the second occurrence of ‘ $x$ ’.

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