

The Reducibility of Relations

HS “Leibniz and the Scholastics”, FS 2018, Philipp Blum, May 14, 2018

The Problem of Structure

Relations have (at least) two essential features properties are lacking: direction and order. If aRb we may both ask whether R holds from a to b or in the other direction and whether R holds of a and b in this or the opposite order. The two questions are different, but correlated. We may choose, without loss of generality, a binary relation $R(x, y)$ as our example. Suppose it holds between a and b , in this order. It follows:

- that the relational fact $R(a, b)$ is *ordered*; it has an internal structure and consists of (at least) two parts, a and b , distinguished by *how* they stand in the relation R : a is R -ing while b is R -ed;
- that, within the relation fact $R(a, b)$, R not simply holds, but holds *in a certain direction*: it holds from a to b , and is thereby different from its converse which would hold from b to a .

The most important specificity of relations, in cases where we have at least a *prima facie* reason to believe in their existence, is their multiple adicity, the fact that they ‘involve’ more than one particular. This gives rise to what I will call “the problem of converses”. The converse of a binary relation is normally taken to be the unique relation that holds between the same particulars in the other direction. There are two roads to its acceptance: the first one starts from the given binary relation R and defines the converse as the unique relation that holds in the other direction – this is the route chosen by Russell and Whitehead in the *Principia*: the converse of R is defined as the unique relation \check{R} which holds between x and y iff R holds between y and x (Russell 1901c: 316, §1.7).¹ This operational conception of converses leaves it open whether the operation $X \mapsto \check{X}$ is total. It may, for example, not be defined for relations without sense or direction.²

A different route starts from the relational fact itself, identifies by analysis the relational properties exemplified and asks about their status. In the same way, it is then argued, the property of being R -related to b is derived from and posterior to the relation R , the property of being such that a is R -related to it is derived from and posterior to the relation \check{R} . Sensible questions may then be asked about the relations R and \check{R} , which may or may not be identical, but are assured to exist in all cases.³

The problem of converses has three aspects:

Problem 1 : ontological profligacy.

“If a book is on a table, *on the table* is a relational character truly predicable of the book. But this is inseparable from another relational character predicable not of the book but of the table. How are these two relational characters connected with each other? We may be tempted to say that the difference between them is purely verbal, so that, whether we

1. Cf. also Schröder (1895: 30), and (Whitehead & Russell 1910: 32). Given the definition of the *Principia*, it can be proved that every relation has a converse (1925: 238–239, *31.13). The same definition of converse was given by 1892: 246.

2. Russell (1901a: 48/307) says that distance is a symmetrical relation without sense.

3. I think this may plausibly be taken to be Russell’s position in the *Principles*: That every relation has a converse is taken by Russell (1903: 25, §28) to be a primitive proposition, where he defines symmetrical relations as those identical to their converses. Russell (1903: 44, §48) uses the same language, saying that in a relational proposition we may regard either one of the terms as the logical subject. It may also be Maccoll’s conception, who calls converse relations ‘reciprocal’ and claims that relations are synonymous with their converses (1902: 359).

say the *book is on the table* or the *table is under the book*, we are merely expressing the same fact in different language. But this cannot be true; for *being on* is different from *being under*; killing from being killed; loving from being loved. Yet it is plain that a single indivisible fact is referred to whether we say that the book is on the table or that the table is under the book.” (Stout 1940: 121)

“...it is hard to see how the state *s* might consist *both* of the relation *on top of* in combination with the given relata and of the relation *beneath* in combination with those relata. Surely if the state is a genuine relational complex, there must be a *single* relation that can be correctly said to figure in the complex in combination with the given relata.” (Fine 2000: 4)

Problem 2 : indeterminacy. How can it be, asked Ramsey (1925: 14, 406), that $(\lambda x(aRx))b$, $(\lambda y(yRb))a$ and $(\lambda x,y(xRy))(a,b)$ represent (are logical forms of) the same proposition, given that they have different components? If they represent the same proposition, and stand for the same fact, however, what are their constituents? If relations are different from their converses, what could give us a reason to take one, but not the other, to be a constituent of a relational fact?

It is not just multiplication of entities that is at stake. Another problem is indeterminacy, both ontological and semantical. Armstrong (1997: 91), e.g., claims that \check{R} is not an increase in being, for every state of affairs containing it is identical with one containing just R . He does not tell us, however, which of the two relations is a constituent of this state of affairs. Williamson (1985) asks us to imagine two languages L' and L'' , both differing from our language L only by inverting the order of arguments following R and by replacing R by its converse \check{R} respectively. By hypothesis, we cannot distinguish between L' and L'' . If relations were different from their converses, we could never distinguish our language from either L' or L'' – we would never be able to know what our relational expressions are standing for. In both cases, the natural reaction is to say that there is no real question because for any relation R , R and \check{R} are identical.⁴ But how can they be identical, if they apply to the same relata only if these are respectively taken to be in different orders?

Problem 3 : regress.

“...when we analyze them, *greater* obviously differs from *less*; thus the two propositions [“*A* is greater than *B*” and “*B* is less than *A*”] seem to be composed of different constituents, and therefore to be necessarily distinct. To deny that they are distinct, it would be necessary to hold that both *greater* and *less* enter into each proposition, which seems obviously false, or else to hold that what really occurs is neither of the two, but that third abstract relation of which Leibniz speaks [...]. In this case, the difference between *greater* and *less* would be one involving reference to the terms *A* and *B*. But this view cannot be maintained without circularity: for neither the greater nor the less is inherently the antecedent, and we can only say that, when the greater is the antecedent, the relation is *greater*, when the less, *less*.” (Russell 1901a: 41/300)

Suppose that Othello (*a*) loves Desdemona (*b*), a fact we may equally well describe as Desdemona’s being loved by Othello. Suppose furthermore that Desdemona does not love Othello, or, what comes to the same thing, that Othello is not loved by Desdemona. Without prejudging questions of identity, we can see these states as ‘arising’ out of the first by inversions of direction (*D*) or of order (*O*).

4. Cf. Williamson (1985: 249) and Armstrong (1978: 42). Williamson’s argument presupposes that relations are individuated by the semantical roles of expressions standing for them.

$$\begin{array}{ccc}
R(a,b) & \xrightarrow{D} & \check{R}(a,b) \\
o \downarrow & & \downarrow o \\
R(b,a) & \xrightarrow{D} & \check{R}(b,a)
\end{array}$$

Because, in this case, love is unrequited, we have non-identities holding on all four sides of the square: Order forces us to distinguish Rab and Rba . Direction forces us to distinguish Rba and $\check{R}ba$. But their interplay forces us to identify Rab and $\check{R}ba$: we have identities along the two diagonals. This diagram thus ‘commutes’, i.e. $O(D(a \rightarrow b)) = D(O(a \rightarrow b))$. I submit that this is explained by the fact that one operation is the converse of the other, ie. only one of order or direction is fundamental. But which one is it? I argue that it is order that explains direction, not direction that explains order.

What conception of R allows us to say that

- = “ Rab ” and “ $\check{R}(ba)$ ” denote the same relational fact;
- pos R holds, but \check{R} does not hold, between a and b (in this order);
- neg \check{R} holds, but R does not hold, between a and b (in this order).

To solve the problem of converses – to explain differential applicability (**pos** and **neg**) without intrinsic directions (\equiv) –, we need to loosen the connection between direction and order.

We may, and must, if the argument from the problem of converses is sound, hold that both “loving” and “being loved” stand for the same relation, even though they apply to their relata in a different order. To do this, we must loosen the connection between relations and direction: even though the ‘directions’ of R and \check{R} are different, this does not distinguish them as relations. The relation, equally well denoted by “ R ” or “ \check{R} ”, is not intrinsically, but only extrinsically directed. In this sense, it is “undirected”, or “neutral” (with respect to direction).

“Neutral relations”, [Fine \(2000: 3\)](#) says, do not hold of their arguments in any specifiable order. Fine’s starting point, as [Castañeda’s](#), is the apparent absurdity of the claim that the fact of a ’s being to the right of b is different from the fact of b ’s being to the left of a . Fine’s conclusion is similar to [Williamson’s](#): we cannot, in general, speak of the “first” and the “second” argument of some relation, identifying these in terms of closeness to the relational expression or their spatial position with respect to it. Similarly, [Fine \(2000: 6\)](#) concludes that “[neutral] relations should [...] be taken to apply to their objects without regard to the order in which they might be given”.

If we give up on the idea, as both [Williamson](#) and [Fine](#) urge, that relations relate their terms in some specific order, how can we then account for their differential applicability, i.e. the fact that the loving relation may hold between [Don José](#) and [Carmen](#) but fail to hold between [Carmen](#) and [Don José](#)? [Fine](#) presents us with two options: positionalism, which reifies argument places and includes them as constituents into relational facts, and anti-positionalism, which takes it to be a brute fact that (some) relations may, when applied to some given terms, yield more than one relational complex.

[Fine \(2000: 9\)](#) construes both these positions as introducing further relata, “for if there were not [any further relata] and if the notion [of exemplification] were indeed order-insensitive, then we would be left with something like the attenuated form of exemplification described above and there would be no way to account for differential application”. This does not follow, however. Positionalism, while committed to argument-places or ‘positions’, can incorporate them into the relation: they can contribute to the ontology of the relational fact not by being further relata related by the relation, but rather by being essential parts of the relation itself. Similarly, anti-positionalism can hold that the similarity between co-mannered

completions (relational facts in which, as a directionalist would say, the relation applies to its arguments in the same order) is extrinsic, but non-relational.

Even amended in this way, Fine's menu of options, however, is not exhaustive: it does not follow from the fact that relations do not exhibit *intrinsic* directionality that they are not directed at all. One and the same relation, $R = \check{R}$, could be extrinsically directed one way when exemplified by Othello and Desdemona in this order and be extrinsically directed another way when exemplified by Desdemona and Othello in this order.

Leibniz

Famously, Leibniz argued in his correspondence with Clarke that relations, if they existed, would be “in two subjects, with one leg in one, and the other in the other, which is contrary to the notion of accidents”⁵

In “De Lingua Rationali”, Leibniz says:

Lingua rationalis ita utiliter constituetur, ut cuilibet vocabulo aliarum linguarum respondens possit, si velimus constitui, v.g. *Titius est magis doctus Cajus*. Sensus est: quatenus Titius est doctus, et Cajus est doctus, eatenus Titius est superior et Cajus est inferior. Haec analysis optima quidem est, sed non exprimitur vis singulorum verborum. Quod ut assequamur, dicendum erit: Titius est doctus et qua talis superior, quatenus inferior qua doctus est Cajus. (Leibniz 1999: 643)

Here, we seem to have a reduction to higher order predication of relations of comparison, and perhaps internal relations more generally. What about external relations? In the “Analysis of Prepositions”, written shortly afterwards, we find:

Accusativus sine praepositione, ut cum verbum activum asciscit accusativum patientis, ut Titius laudat Cajum, seu Titius laudat, quatenus Cajus est patiens. (Leibniz 1999: 652)

What about “quatenus”? Jauernig (2010: 202) interprets it as a relation:

The reality of relations of connection, by contrast, depends on God's will, because they obtain in select possible worlds as a result of God's (possible) free decrees. [footnote omitted] That is, ‘et eo ipso’ can be read as short for ‘and by that very fact, due to God's free decrees’. [footnote: “Similarly, ‘quatenus’ can be read as short for ‘insofar as, due to God's free decrees’.” In understanding Leibniz's non-truth-functional connectives in this way, I am disagreeing with (among others) Burdick who claims that ‘et eo ipso’ brings the sentences ‘David is a father’ and ‘Solomon is a son’ “together mentally, i.e., without any additional ontological commitments” (“Leibniz's Problem,” 10). On my view, ‘et eo ipso’ brings the two sentences together via God's special decrees, which amounts to an additional ontological commitment. The latter assessment is supported by how Leibniz measures the perfection of worlds. The perfection of a world is determined by how much “being” or “essential reality” it contains, by how much variety there is, and by the simplicity of its order and laws.”] These decrees can be understood either as concerning certain (contingent) laws that govern the relations in question, such as, for instance, the decree of the laws of molecular genetics that govern family relations, or, more commonly, as concerning specific relations between particular things, such as, for instance, the decree that Caesar is spatially located on top of, or in the Rubicon at a certain time.

5. “...en deux sujets, qui auroit une jambe dans l'un, et l'autre dans l'autre, ce qui est contre la notion des accidens.” (Leibniz's fifth letter to Clarke, 1890: 401 (translations: 1956: 71 and 1989: 339)).

In “Grammaticae cogitationes”, we find:

Optime sic explicabitur ut Paris est amator Helenae, id est: Paris amat *et eo ipso* Helena amatur. Sunt ergo duae propositiones in unam compendiose collectae. Seu *Paris est amator, et eo ipso Helena est amata. Ensis est ensis Evandri*, id est *Ensis est supellex, quatenus Evander est dominus. Poeta est lectus quatenus ille vel ille est legens*. Nam nisi obliquos casus resolas in plures propositiones nunquam exhibis quin cum Jungio novos ratiocinandi modos fingere cogaris. Vel sic: *Paris amat Helenam* id est [*Paris*] *putat quod Helena est sibi futura jucunda. Ensis Evandri*, id est *Ensis qui est subditus quatenus Evander est dominus*. Subditum vel subditus est quod patitur quatenus alius agit, et quatenus is est justus. (Leibniz 1999: 115)

We may, following Leibniz, describe the relational complex on this non-fundamental level of analysis by “Othello loves in so far as Desdemona is loved” or “Othello loves and eo ipso Desdemona is loved”. This analysis has three parts:

- (i) it ascribes to Othello the relational property of loving Desdemona and the non-relational property of loving (ie. loving someone, being a lover);
- (ii) it ascribes to Desdemona the relational property of being loved by Othello and the non-relational property of being loved (ie. being loved by someone, being someone beloved);
- (iii) it ascribes to the facts stated by (i) and (ii) the relation making true claims such as “*p* in so far as *q*” or “*p* and eo ipso *q*”.

This three-pronged analysis allows us to keep what is right in the alternative accounts:

- In this sense of (i) and (ii), we may say, with Fine, that the asymmetric relation *R* distinguishes between two parts of the sum that exemplifies it by coordinating them with different things, e.g. lovers and beloved ones. This allows us to say that Othello, Don José and Abelard, say, have something in common: they are lover parts of fusions exemplifying the neutral amatory relation.
- In this sense of (i) and (ii), we may say, with the positionalist, that this difference between Othello and Desdemona, as parts of the relational complex, is due to their playing different rôles, ie. entering into this complex in different ways – as lover and as beloved respectively.
- The “*p* in so far as *q*” and “*p* and eo ipso *q*” locutions express that “*p*” and “*q*” have the same fundamental truthmaker.⁶

But what *is* the EO IPso? If we had universals and exemplification would be partial identity, we would have a nice theory:

“Abelard insofar as he loves Heloise is partially identical with Loving, in virtue of being partially identical with Loving-by. Heloise insofar as Abelard loves her is partially identical with Loving in virtue of being partially identical with Loving-of. In general the blanks filled by noun phrases in relation predicates, if they correspond to anything, will correspond to aspects of a relation. [...] Notice that, on this account there is a necessary connection between Abelard insofar as he loves Heloise and Heloise insofar as Abelard loves her. Neither aspect can exist without the other. This connection is due to the fact that necessarily, *Abelard* loves *Heloise* if and only if *Abelard* loves *Heloise*. (Baxter 2001: 457-8)

If Heloise also loves Maria, however, all *three* of them turn out identical!

6. Mates (1986: 216) is right about this, though wrong in taking truthmaking to be implication (or rather: entailment): “[Paris is a lover, and *eo ipso* Helen is a loved one”] tells us that those “facts” or individuals-cum-accidents that make “Paris is a lover” true also make “Helen is a loved one” true; presumably, if those facts were more narrowly described, the resulting propositions [...] would actually imply that Paris loves Helen.”

References

- Armstrong, David M., 1978. *A Theory of Universals: Universals and Scientific Realism, Volume II*. Cambridge: Cambridge University Press.
- Armstrong, David M., 1997. *A World of States of Affairs*. Cambridge: Cambridge University Press.
- Baxter, Donald L.M., 2001. Instantiation as Partial Identity. *Australasian Journal of Philosophy* 79(4): 449–464.
- Fine, Kit, 2000. Neutral Relations. *The Philosophical Review* 109(1): 1–33.
- Jauernig, Anja, 2010. Disentangling Leibniz’s Views on Relations and Extrinsic Denominations. *Journal of the History of Philosophy* 48(2): 171–205.
- Johnson, W.E., 1892. The Logical Calculus. II. *Mind* 1(2): 235–250.
- Leibniz, Gottfried Wilhelm, 1890. *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, volume 7. Berlin: Weidmannsche Buchhandlung. Edited by C.I. Gerhardt; reprint: [Leibniz \(1965\)](#).
- Leibniz, Gottfried Wilhelm, 1965. *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*. Hildesheim: Georg Olms Verlagsbuchhandlung. Reprint of the Gerhardt edition of the philosophical papers in 7 volumes.
- Leibniz, Gottfried Wilhelm, 1989. *The Principles of Philosophy, or the Monadology*. Indianapolis, Indiana: Hackett Publishing Co. Edited by Roger Ariew and Daniel Garber.
- Leibniz, Gottfried Wilhelm, 1999. *Philosophische Schriften: 1677 – Juni 1690, Teil A, B, C, D*. Number VI,4 in Akademie-Ausgabe, Berlin: Akademie Verlag.
- Leibniz, Gottfried Wilhelm & Samuel Clarke, 1956. *The Leibniz-Clarke Correspondence*. Manchester: Manchester University Press. Edited by Hubert G. Alexander.
- MacColl, Hugh, 1902. Symbolic Reasoning IV. *Mind* 11(43): 352–368.
- Mates, Benson, 1986. *The Philosophy of Leibniz: Metaphysics and Language*. Oxford: Oxford University Press.
- Ramsey, Frank Plumpton, 1925. Universals. *Mind* 34(136): 401–417. Reprinted in [Ramsey \(1931: 112–137\)](#) and in [Ramsey \(1978: 17–39\)](#), cited after reprint in [Ramsey \(1990: 8–30\)](#).
- Ramsey, Frank Plumpton, 1931. *The Foundations of Mathematics and other Logical Essays*. London: Routledge & Kegan Paul. Edited by R.B. Braithwaite.
- Ramsey, Frank Plumpton, 1978. *Foundations: Essays in Philosophy, Logic, Mathematics and Economics*. London: Routledge & Kegan Paul. Edited by D.H. Mellor.
- Ramsey, Frank Plumpton, 1990. *Philosophical Papers*. Cambridge: Cambridge University Press. Edited by D.H. Mellor.
- Russell, Bertrand Arthur William, 1901a. On the Notion of Order. *Mind* 10(37): 30–51. Reprinted in [Russell \(1993: 291–309\)](#).
- Russell, Bertrand Arthur William, 1901b. Recent Work on the Principles of Mathematics. *The International Monthly* 4: 83–101. Republished as “Mathematics and the Metaphysician” in [?](#): 74–96.
- Russell, Bertrand Arthur William, 1901c. Sur la Logique des relations avec des applications à la théorie des séries. *Rivista di matematica* 7(1–3): 115–148. Translated by R.C. March, with corrections by Russell, as [Russell \(1956\)](#) and reprinted in [Russell \(1993: 314–349\)](#).
- Russell, Bertrand Arthur William, 1903. *The Principles of Mathematics*. Cambridge: Cambridge University Press. 2nd edition: [Russell \(1937\)](#).
- Russell, Bertrand Arthur William, 1937. *The Principles of Mathematics*. 2 edition. London: George Allen & Unwin.
- Russell, Bertrand Arthur William, 1956. The Logic of Relations. In *Logic and Knowledge: Essays, 1901–1950*, pp. 1–38. London: George Allen & Unwin. Translation of [Russell \(1901b\)](#).
- Russell, Bertrand Arthur William, 1993. *Towards the Principles of Mathematics*. Number 3 in The Collected Papers of Bertrand Russell, The McMaster University Edition, London: Routledge. Edited by Gregory H. Moore.
- Schröder, Ernst, 1890. *Vorlesungen über die Algebra der Logik (Exakte Logik)*, volume 1. Leipzig: B.G. Teubner. Reprint in [Schröder \(1966\)](#).

- Schröder, Ernst, 1891. *Vorlesungen über die Algebra der Logik (Exakte Logik)*, volume 2. Leipzig: B.G. Teubner. Reprint in [Schröder \(1966\)](#).
- Schröder, Ernst, 1895. *Vorlesungen über die Algebra der Logik (Exakte Logik)*, volume 3. Leipzig: B.G. Teubner. Reprint in [Schröder \(1966\)](#).
- Schröder, Ernst, 1905. *Vorlesungen über die Algebra der Logik II, 2. Abteilung*. Leipzig: B.G. Teubner. Herausgegeben von E. Müller, wiederabgedruckt in [Schröder \(1966\)](#).
- Schröder, Ernst, 1966. *Vorlesungen über die Algebra der Logik*. New York: Chelsea. Reprint of [Schröder \(1890, 1891, 1895, 1905\)](#).
- Stout, George F., 1940. Things, Predicates and Relations. *Australasian Journal of Psychology and Philosophy* 18(2): 117–130.
- Whitehead, Alfred North & Bertrand Arthur William Russell, 1910. *Principia Mathematica*, volume 1. Cambridge: Cambridge University Press. 2nd ed. : [Whitehead & Russell \(1925\)](#), abridged as [Whitehead & Russell \(1956\)](#).
- Whitehead, Alfred North & Bertrand Arthur William Russell, 1912. *Principia Mathematica*, volume 2. Cambridge: Cambridge University Press. 2nd ed. : [Whitehead & Russell \(1927a\)](#), abridged as [Whitehead & Russell \(1956\)](#).
- Whitehead, Alfred North & Bertrand Arthur William Russell, 1913. *Principia Mathematica*, volume 3. Cambridge: Cambridge University Press. 2nd ed. : [Whitehead & Russell \(1927b\)](#), abridged as [Whitehead & Russell \(1956\)](#).
- Whitehead, Alfred North & Bertrand Arthur William Russell, 1925. *Principia Mathematica*, volume 1. 2 edition. Cambridge: Cambridge University Press. 1st ed. : [Whitehead & Russell \(1910\)](#), abridged as [Whitehead & Russell \(1956\)](#).
- Whitehead, Alfred North & Bertrand Arthur William Russell, 1927a. *Principia Mathematica*, volume 2. 2 edition. Cambridge: Cambridge University Press. 1st ed. : [Whitehead & Russell \(1912\)](#).
- Whitehead, Alfred North & Bertrand Arthur William Russell, 1927b. *Principia Mathematica*, volume 3. 2 edition. Cambridge: Cambridge University Press. 1st ed. : [Whitehead & Russell \(1913\)](#).
- Whitehead, Alfred North & Bertrand Arthur William Russell, 1956. *Principia Mathematica to *56*. Cambridge: Cambridge University Press. Abridged second edition ([Whitehead & Russell 1925, 1927a, b](#)); first edition: [Whitehead & Russell \(1910, 1912, 1913\)](#).
- Williamson, Timothy, 1985. Converse Relations. *The Philosophical Review* 94: 249–262.