What singletons could be

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comments welcome

"Alas, the notion of a singleton was never properly explained." (Lewis 1991: vii)

Abstract

According to David Lewis (1991), membership is the only mystery set theory has left for us mereologists. In fact, it is only the special case of membership-in-a-singleton that troubled Lewis and needs to trouble us, if we take the subset relation to be a special case of the (antecedently understood) relation of part to whole. My task, then, is to explain what it means to say that a thing, a, is a member of a special sort of set, $\{a\}$, having a as its only member. My proposal, roughly, is that singletons, and sets in general, are a special sort of *property*, i.e. monadic universals in a broadly Armstrongian sense. $\{a\}$, in particular, is the property *having all the properties of* a, and membership is the relation of exemplifying such properties. I will call such properties "natures", the general idea being that they provide enough structure to enable mereology to go proxy for set theory. I will sketch, develop and make plausible my proposal and then defend it against its two main alternatives, developed by Armstrong and Bigelow.

What are sets? Sets are things which have members.[1] Sets depend for their existence on the existence of their members. The converse, however, seems disputable: although I necessarily exist iff my singleton does, it certainly sounds weird to say that I depend on my singleton for my existence.[2] According to standard iterative set-theory, sets are "built" out of their members. So we have to understand what this building process amounts to if we want to find out what sets are. Thanks to Lewis' book *Parts of Classes* (1991), the problem reduces itself to a special case: what are singletons and how do they relate to their members? What is the difference between an object a which is not a set (an 'urelement' or 'individual' as we will call it) and its singleton $\{a\}$? My claim is that this difference is that between a thing and its properties: $\{a\}$ is what I will call the *nature* of a, the property exemplification of which makes a the thing it is.

If we knew what singletons were, we could then construe sets, and classes in general, as mereological sums of their singleton subclasses, for we know what a mereological sum is if we know what its parts are (or so we shall suppose for the time

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[1]Some terminology: In the following, I will call anything which has members a "class". Sets are classes that are members themselves. An individual is anything which is not a class, i.e. which is a member, but does not have any members.

[2]This, at least, is Kit Fine's intuition of which he has made a criticism of the standard modal account of essence (1994: 5).

being). My thesis is that $\{a\}$ is a certain property of a, in the most general and most inclusive sense of all of a's properties. To clarify and make plausible my proposal, I first have to introduce properties and what I will call "natures". Then we can tackle the central problem of exemplification and deal with membership as a special case. Exemplification being clarified, I will state my thesis and finally defend it.

1 Properties and Natures

I propose and will defend below a variant of the following contextual definition:

1.1 $\ldots \in \ldots \quad := \quad \ldots$ *is determined by* $\ldots$

Membership is a relation between an individual and a class. Exemplification, of which determination is a special case, is the relation between a (which is either a particular or a property) and F (which has to be a property) which holds iff a is F. I want to identify classes with a special sort of property,[3] i.e. fusions of what I call "natures". My project presupposes that it makes sense to speak of 'universals' in a broadly Armstrongian sense.[4] Universals do not have to be ontologically fundamental, however, for my proposal to go through.[5] Whatever their exact nature, however, I pose universals to be nonlinguistic and sparse. Thus they differ from predicates and from the less basic other properties 'built' in some way out of them. Natures, the kind of universals that I want to identify with classes, are closest to the categorical gulf separating (multiply occurring) universals from (unrepeatable) particulars: although they can, in principle, be exemplified repeatedly, they determine *all* the qualitative features of *one* entity. They are thus never exemplified by *different* things: any things sharing them are ipso facto indiscernible.

What, then, are natures? a's nature is the property *being (qualitatively) indiscernible from a* ($\lambda x(x \cong a)$), composed mereologically out of all of a's properties. To be indiscernible from a, b has to share all of a's properties and to lack the others which a does not have. So *being indiscernible from a* simply is *having all and only the properties of a* ($\lambda x(Fa \leftrightarrow Fx)$). On the face of it, this seems to be a second-order property, quantifying over the first-order properties of a.[6] Though I grant this for the "and only" part, I want to deny it for "all": Armstrong's theory is one of *sparse* universals, resting on the assumption that the qualitative features of our world can be wholly captured by a determinate (and perhaps even finite) list of fundamental properties. Natures, then, are basically first order: they are (mereological) fusions of all properties of one thing, whatever these in turn happen to be. By "being indiscernible from a" I do not mean the 'property' (if it is one) *being identical to a*, unless identity is understood as qualitative, rather than numerical. By discernibility, I mean discernibility *by properties*:

1.2 *(Diversity of Discernibles) If two things are discernible, they differ in at least one property.*[7]

[3]My proposal may seem mistaken for a lot of reasons, the most prominent being that classes are, on the face of them, particulars. My answer is that appearances are misleading.

[4]Here and in the following I mean by "particular" anything that isn't capable of multiple exemplification, by "universal" anything that is.

[5]A trope theorist, e.g., could well accept it, as long as he takes monadic universals to be, e.g., fusions and not classes of tropes.

[6]A distinction of Bigelow's (1993: 94) will prove useful: A *second-degree property* is a property of properties; a *second-order property* is a property of particulars, but one which is entailed by a property of their properties. *Being a colour* is a second-degree property, *being colored* one of second order.

[7]The converse of (1.2) holds by definition: Differing in at least one property is just what makes

Universals, in Armstrong's sense, are a special sort of thing: they are capable of occurring repeatedly, they are wholly present in all their exemplifications and wholly 'qualitative' (Armstrong 1978a: 110, 1989a: 106): There is nothing that could distinguish two universals sharing all their properties. With respect to particulars, however, identity of indiscernibles is a substantive metaphysical claim.[8]

Universals differ from tropes by being strictly identical in different exemplifications. Although they are not, like tropes, dependent on a specific particular, universals are dependent on there being some particular exemplifying them. I thus construe them on an Aristotelian, *in re* model:

1.3 *All (actual) universals are exemplified.*[9]

Natures have some peculiar features. A particular's nature not only consists of all its properties, but it also does not consist of anything else. To be indiscernible from a, a particular b not only has to share all of a's properties, but it also has to *lack* all the properties a does not exemplify.[10] Take, as an example, a particular a having properties F, G and H and suppose this to be a complete description of a's nature. The fusion $F \oplus G \oplus H$ then stands in a certain relation T to the second-degree property *being a property of a*.[11] Thus the fusion has the second-degree property *standing in the T relation to* being a property of a. From this second-degree property, we get a second-order property of a itself, namely *having F, G, H as the only properties*, which is a's *totality property*. This is a fourth property of a and a property that a could have lacked. If *having F, G, H as the only properties* is a further property of a, shouldn't we have included it from the outset (thus embarking on a regress)? No, because second-order properties of a particular do not have to be included in a complete description of it. Totality properties are unique: a's fourth property is not part of the fusion that stands in the T relation to *being a property of a*; there is no way therefore to include it in the collection of those properties of which it is said that they suffice for a complete description of a.[12]

We postulate universals in order to have a sparse theory of properties. We want to have such a theory to demarcate 'unified' particulars from gerrymandered ones, rabbits and the fusion of my nose and the Eiffel tower from all these indefinitely many parts and fusions of our 'elite' objects we have never cared to think about.

two things discernible. From (1.2) it follows, according to Leibniz' Law, that discernibles are not identical.

[8]In the theory of exemplification I favour, which makes exemplification a special case of parthood, this difference could be explained as follows: Because (at least some) particulars have other than non-spatiotemporal parts, they can differ while sharing all their universals. Universals, however, are wholly non-spatiotemporal, in the sense of being fusions of non-spatiotemporal parts of particulars. It is possible and plausible, however, to locate universals where their members are (on the basis of 1.3). Although they are non-spatiotemporal, then, they are contained in that big particular which is space-time itself.

[9]What (1.3) amounts to, of course, will heavily depend on what counts as a case of exemplification. Armstrong, e.g., allows for past and future exemplification, but not for exemplification by merely possible things. Presentists could like to go further and maintain that the nature of Socrates ceased to exist with Socrates' death. For myself, I allow even for exemplification by merely possible things.

[10]Here I am borrowing from Armstrong's account of totality facts (1989: 95-96, 1997: 199-200), although he presumably would deny that it also applies to natures. While he does not mention totality facts in his account of natures, in (1997: 35) he considers the possibility that *methane* could contain a negative element of the sort *having hydrogen atoms as constituents that are not bonded to any other atoms than to the constituent carbon atom.*

[11]Armstrong calls T an "alling" or "totalling" relation (1997: 199), which is somehow misleading, as it does not hold, as we will see, between *being a property of a* and *all* of a's properties.

[12]The qualms of those that smell a rat here are more adequately met by not describing T as a "totalling" relation - it is, in any case, not a relation between *all* of a's properties and *being a property of a*. The distinction between properties had by a and properties that have to be mentioned in a complete description of a is better illustrated with fusions: if $F \oplus G$ is a property, as will be argued below, then this property does not have to be included in the complete description either.

Some particulars are *by (their) nature(s)* more eligible referents than others; some coexemplifications of properties give better demarcated particulars than others. It is therefore useful to speak of 'thick' and 'thin' natures. The nature of a particular rabbit, e.g., is thick: it is in virtue of being that particular rabbit that a certain entity stands in a huge amount of causal and spatiotemporal relations. The nature of the fusion of my nose and the Eiffel tower, on the other hand, is comparatively thin (though thicker than many other natures): the causal powers bestowed by it dissipate into those bestowed by *being my nose* on my nose and those bestowed by *being the Eiffel tower* on the Eiffel tower. By the sole fact that there is a fusion of their natures, the fusion of my nose and the Eiffel tower does not gain much causal power not attributable to one of its two components. We can describe it, like many other fusions postulated by (2.1), (almost) completely in terms of properties of the form *having an F part.*

2 Mereology

In the following, I take for granted some mereological principles. One of them is "Unrestricted Composition":

2.1 *If there are some things, they have a mereological fusion.*

By the following "Uniqueness of Composition" principle fusion, which I denote by "$\oplus$", is functional:

2.2 *Only one (actual) whole is composed of some given (actual) parts. Of all the different ways in which some parts can form a whole, at most one is actual.*

(2.1) is ontologically acceptable because any given whole supervenes on the arrangement of its parts, because mereology is, in other words, 'ontologically innocent':

2.3 *Fusions are nothing over and above the parts they are composed of.*

(2.3) justifies the following comprehension schemata, which hold of all things x, y and z and all properties P and allow us to include properties of parts into the nature of their whole and conversely to account for things which are best described as parts of other things:

(1) $\exists \phi \,\Box\, \forall x, y, z(x = y \oplus z \;\Rightarrow\; (y \text{ has } P \Leftrightarrow x \text{ has } \phi))$

(2) $\exists \psi \,\Box\, \forall x, y, z(x = y \oplus z \;\Rightarrow\; (x \text{ has } P \Leftrightarrow y \text{ has } \psi))$

(1) and (2) give us properties of the form *having an F part* ($\lambda x\ \exists y, z\ (x{=}y{\oplus}z{\wedge}Fy)$) and *being part of an F* ($\lambda x\ \exists y\ F(x{\oplus}y)$) and thus allow us to ascribe properties of parts to their wholes and properties of wholes to their parts.

We can thus explain another peculiar feature of natures: the fact that they are maximally mereologically specific. A property F is maximally mereologically specific (specific for short) iff anything a exemplifying it is such that neither any proper part of a nor any whole of which a is a proper part exemplify F.[13] To what degree a property is specific is an objective, substantial second-order property, which is independent of our ways of counting things and the stock of predicates provided by our language. By (1) and (2), natures are a limiting case of specific properties. If F is the nature

[13] We could call a property F upwards specific if it is not the case that, if it is had by an object, it is also had by some object y of which x is a part; F is downwards specific if it is not inherited by any of the parts of an object that has it. The dual of downwards specificity (i.e. being such that all parts inherit it) has been called "dissective" by Casati and Varzi, following Goodman, and "cumulative" by Simons (1987: 111).

of a, it could not have been a property of any proper part of a (for, by (1) it includes a property for any part of a).[14] F could neither have been the nature of $a \oplus b$, for the latter will have properties (e.g. *being composed of an a and a b part*) a lacks. It could not even have been a *property* of $a \oplus b$, as it includes a 'totality' property with respect to a which $a \oplus b$ lacks.

Do our comprehension schemata multiply universals beyond necessity? In a sense they do, for *having an F part* is not the same property as F.[15] In another sense they do not, for anything that justifies us in applying "F" to x *ipso facto* justifies us in applying "has an F part" to $x \oplus y$. By Hume's principle ('no necessary connections between distinct existents'), this gives us a reason to suspect that F and *has an F part* are not wholly distinct, i.e. that properties of parts are somehow 'included' in properties of their wholes. Natures allow us to be more precise: If a is F, but $a \oplus b$ is not, the natures of a and $a \oplus b$ differ in that the one includes F and the other *having an F part.* We can thus distinguish two ways a property F can be part of a nature: F is *included* in N_a iff a has a totality property which entails it; it is (merely) *contained* in N_a iff a has a proper F part but is not F. This brings out the sense in which *having an F part* is, ontologically speaking, nothing over and above F, although the corresponding predicate ascribes F 'obliquely' as it were: Whether or not something has an F part depends only on what is an F. I take this feature of the logic of properties to be closely connected to Unrestricted Composition: If (2.1) really were as problematic as its enemies suppose it to be, we would expect to get entities exemplifying new properties. Although this is true in a sense, (1) and (2) show that the new properties are nothing but variants of those we already had.

As I accept Unrestricted Composition (2.1) and assume that there are properties, I have to explain what it means for something to be the fusion of some properties and, in particular, what exemplifies the fusion $F \oplus G$ of the properties F and G. The idea is that the exemplification of $F \oplus G$ by c determines a particular partition of c in parts exemplifying F and G respectively. Fusions of properties account not necessarily for similarity of simples, but for similarity of *wholes.* My proposal for fusions of properties is thus the following:

2.4 *$F \oplus G$ is exemplified by c iff* $c = a \oplus b \wedge ((Fa \wedge Gb \rightarrow (Ga \vee Fb)) \rightarrow (a \cong b \wedge Fa \wedge Ga))$

So c exemplifies $F \oplus G$ iff either it is both F and G, if it has indiscernible proper parts which are both F and G or if it has proper parts a and b such that a is F but not G and b is G but not F. For natures, we can go along with a simplified version:

2.5 *If F and G are natures, $F \oplus G$ is exemplified by c iff* $c = a \oplus b \wedge Fa \wedge Gb$

The fusion of some natures is thus a property of the fusion of the particulars exemplifying them. As natures are specific, no fusion of different natures is a nature. This may explain why it is so easy to describe gerrymandered fusions of particulars.[16] Any particular exemplifying the fusion $F \oplus G$ of natures F and G must have parts, namely at least one exemplifying F and one exemplifying G. If $F \neq G$, these parts therefore must be proper.

[14] As we will see later, even bare particulars would have a nature.

[15] Though I allowed for improper parts and thus guaranteed the truth of "Every F has an F part", the converse fails for upwards specific (i.e. most) properties.

[16] Although the fusion of a cat a and a dog b is neither a cat nor a dog, we know a priori (by (1)) that it is partly cat and partly dog. If we know the nature of $a \oplus b$, we know the nature of its parts - but only if we know how a and b have been put together in $a \oplus b$.

3 Exemplification and Determination

Armstrong defends a theory of "immanent" universals. What does immanence amount to? My proposal is, in short, that it amounts to properties being parts of their exemplifying particulars.[17] Exemplification, then, is just parthood:

3.1 *A universal is a (nonspatiotemporal) part of every particular that exemplifies it.*

Whenever a exemplifies F, two relational properties are exemplified by a and F respectively, namely *having* F *as a property* and *being a property of* a. The first of these just mimicks F. The latter, however, differs from F in important ways: whenever it is had by a property G, it is an essential property of G.

Whenever we wonder whether F is an essential property of a particular a, we consider counterparts of a in other possible worlds and ask whether these are F. Whatever your favourite account of the counterpart relation, brute identity will not do, for it would turn a into a universal. Properties, however, are identical in all their exemplifications and thus identical even across possible worlds. Take a merely possible particular a in a non-actual possible world w and assume it is F (there). Then *being exemplified by* a is a property of F *already* in our actual world. There is no possible world where F lacks it. By (1.3), for a property to exist 'in' a world is for it to be exemplified there. So in every world where F exists, it has the property of *being exemplified by* a. If F would be exemplified by something (actual or possible) which is not F (in its world, of course), it would be a different property, necessary coextensiveness being at least a necessary condition of property identity. Thus we have a modal asymmetry between two necessarily coexemplified relational properties.

Exactly the same happens with membership: Whereas it follows from the existence of both the singleton and its member that, necessarily, the other relatum exists, only the singleton is essentially the singleton of its member but the member is not essentially the member of its singleton (see Fine 1994: 5).[18] This asymmetry is based on the difference between an attributive and a referential reading of the condition that defines a set:

> "Although it surely is *not* necessary that Cicero is an orator, it *is* necessary that Cicero is a member of that set whose members happen to be all actual orators." (Jubien 1981: 172-173)

On an attributive reading of its defining clause, $\{x \mid x \text{ is an orator}\}$ might have had other members than it actually had. Read referentially, however, it is necessarily the set of all and only the orators.[19] Likewise, there is something like an attributive and referential reading of predicates: we can take "F" to denote whatever all and only the Fs there are have in common. Then nothing other than *these* Fs could have had *this* property. More often, however, we individuate properties attributively, e.g.

[17] This claim needs qualification. Exemplification is parthood *tout court* in the case of intrinsic properties. Extrinsic properties are parts not of the particular they are ascribed to but of a more inclusive whole, maximally the world which contains the particular. As I do not have the space to elaborate on it, however, I will ignore such complications in the following and just assume that universals are intrinsic to all their exemplifications.

[18] In (1981: 179), Fine has called the first of these phenomena "rigidity of membership": "... it is essential to the identity of a set that it have the members that it does."

[19] In (1977: 141) Kit Fine drew an important distinction between two ways sets of possible worlds can be evaluated. This distinction is best brought out by considering the set of all possible worlds. Viewed as a proposition, Fine says, this set exists necessarily: whatever our possible worlds are, they necessarily form a set (or a proper class for that matter). Viewed as a set, however, its existence depends essentially on the existence of each possible world. If their existence were contingent, then the existence of their set would be so too. My point here is that natures behave like sets and unlike propositions.

by causal roles. Then it is in most cases contingent whether or not some a has a property with this role.

I call 'determination' the special case of exemplification where the exemplified property determines *all* qualitative features of the exemplifying thing. Every determining property entails all the properties exemplified by whatever exemplifies it:

3.2 *G determines a* $:\Longleftrightarrow \forall F\ (Fa \to G \Vdash F)$

Necessary coextensiveness being at least a necessary condition on property identity, "$G \Vdash F$" implies that every G is F. I do not wish to commit myself to any particular account of property identity, so I will simply stipulate that entailment between property predicates (i.e. inclusion of their extensions and whatever else might be needed) gives us a partial (reflexive, antisymmetric and transitive) order on properties. Given (3.1), it seems tempting to analyse property entailment as parthood: $\forall x(Gx \to Fx)$ would then follow from $G \Vdash$ by the transitivity of parthood.

Determination is monotone: if G determines a, any property having G as a part also determines a. The least thing determining a is a's nature. Only natures or fusions of natures can determine something: so determination is just the restriction of exemplification to natures.[20] Property entailment $\Vdash$ being only defined for properties, no 'mixed' fusion of a property and a particular can determine anything. Any fusion of natures determines a definite number of things. Let F be such a fusion, determining a. Then $F = A \oplus G$, A being the nature of A. Either $A \Vdash G$ or not. In the first case, $F = A$ and we are done. In the second case, there is a b determined by G. So $G = B \oplus H$ and we iterate the procedure.[21]

4 Singletons of particulars

$\{a\}$, according to Lewis, is mereologically atomic, whereas the nature of a has the properties of a as parts. I deny, therefore, Lewis' 'Main Thesis', i.e. that the parts of a class are all and only its subclasses. Its relevant half, the 'Second Thesis' (that no class has any part that is not a class) follows from the following premises:

First Thesis:	One class is a part of another iff the first is a subclass of the second.
Division Thesis:	Reality divides exhaustively into individuals and classes.
Priority Thesis:	No class is part of any individual.
Fusion Thesis:	Any fusion of individuals is itself an individual.

Based on (3.1), I deny the Priority Thesis.[22] The fact that Lewis', but not our singletons, are atomic, does not really make a difference, because what matters is uniqueness of decomposition: any class has to be decomposable uniquely into a definite number of members. Lewis gets this feature by the converse of (2.2), standardly accepted in extensional mereology. We, on the other hand, get the same by the

[20] Suppose a is determined by F. F must contain a's nature, say A, so $F = A \oplus G$. From $(A \oplus G)a$ it follows by (2.4) that $a = b \oplus c$, Ab and Gc. If c is indiscernible from b, then G is part of b's and thus of a's nature. Otherwise, G must be the nature of c. For suppose it is not. Then there is a property B of c which is not entailed by G. But then a has a property, namely *having a B part*, which is not entailed by G. So G does not determine a.

[21] In particular, fusions of the things determined by a fusion of natures A exemplify A, but they are not determined by A, for they have, by (1) more properties than the sum of the properties of their parts.

[22] We have to distinguish between two readings of the Division Thesis: (1) $\forall x$(x is either a class, an individual or a fusion of an individual and a class) and (2) $\forall x$(x is part of either an individual, a class or a fusion of an individual and a class). Properties that are not natures and do not include natures do not have members, on my account. If we accept them as individuals (which seems the natural option of an Armstrongian theory), we can accept both readings. Because of the empty set, not all individuals are particulars anyway. Even if we do not and reject (1), (1.3) commits us to (2), for any property is part of at least one nature.

specificity of natures and our account of property fusion. Our singletons, although mereologically complex, are relatively atomic in the sense that they have no proper parts that are classes. But if b is a proper part of a, is not then the nature of b part of the nature of a? No, for it includes a totality property which a lacks. So no classes are part of a singleton.

What about the empty set? It is an individual, for it cannot have members. It is part of any nature, though not part of everything there is.[23] Lewis choose the fusion of all individuals to play the role of the empty set. This, however, makes it mysterious why it is part of all classes. The empty set, in our account, is what all natures have in common, namely a property had by everything whatsoever. We can, if we want, call this universal property *existence*. Why cannot the empty set be the nature of anything? Because natures, as we have seen, include 'totality properties'. Even the natures of bare particulars, if there are any, are therefore 'thicker' than the null set is, for they cannot have *existence* as their only property, without thereby also having *having no other properties than existence* as another, different property.

Although the empty set is an individual, it is not an ordinary one. In the iterative conception of set theory, it is not 'given', like the urelements, at the first, but 'formed' at the second stage, as the 'collection' of all urelements in the case there are none. This is not elegant, but it brings out why it may seem weird to call $\emptyset$ an individual. I think we can mitigate these qualms: although $\emptyset$ is an individual, it is not a particular, but a property.[24] Treating $\emptyset$ as *existence*, we can explain why it is *part* of every class, for nothing non-existent can have a nature.[25]

5 Singletons of classes

Not only particulars, but also natures have properties and thus natures. Normally, the nature of a nature will include properties such as *being exemplified by a* not exemplified by the particular it determines.[26] By (3.1), the nature of the nature of a is a part of the nature of a and a part of a, because it is exemplified (as a second-degree property) by the nature of a and (as a second-order property) by a. The cumulative hierarchy thus becomes a $\subset$-ordered chain of smaller and smaller parts of the urelements we started which. Pure sets are parts of *existence*, i.e. nonspatiotemporal parts of everything there is.

I now have to show that my account of $\in$ gives us a satisfactory reading of $\subset$ which we took to be a special case of the part/whole-relation. Whatever account of $\subset$ we are willing to give, the following had better be true of it:

(3) $A \subset B \iff \forall x (x \in A \Rightarrow x \in B)$

[23] Following Lewis (1991: 11) and all mereologists except two, I deny that there is a mereological 'null-part', which is part of every thing. Even if $\emptyset$ is part of every class, it is not part of everything. What is unacceptable about the null individual o is that, for anything a, $a - o = a$. If a is an individual, however, it is "wellnigh unintelligible how anything could behave as the null individual is said to behave" (Lewis 1991: 11). The subtraction of *existence* from a, however, gives us a, if a is a class, or something that is not a class, for, by (1.3), nothing that does not contain *existence* can be the nature of anything.

[24] My proposal also has the slight advantage of alleviating those who, like Potter, see Lewis' choice of the fusion of all individuals as "evidently one of those technical tricks one uses to prove technical results" (1993: 363) and consider it possible that nothing exists. If it were possible that there is nothing, *existence* would still be available, though uninstantiated in one possible world.

[25] I do not think that this point is very important. It simplifies Lewis' disjunctive definitions but it does not depend on or imply my construal of sets as (fusions of) natures.

[26] I say "normally" for non-wellfounded sets do not seem to me to be *excluded* by our account. On the construal of Platonic forms as self-exemplifying paradigm properties, e.g., they would be self-singletons, i.e. of the form $a = \{a\}$. Whether or not the form Beauty is itself beautiful should not be decided on mathematical grounds alone.

Interpreted in terms of exemplification, (3) says that whatever is determined by A is determined by B. $\subset$ thus corresponds to property entailment, $\Vdash$, restricted to (fusions of) natures. $\subset$ is a partial order because property entailment is.[27] So two fusions of natures A and B are identical iff $A \Vdash B$ and $B \Vdash A$, i.e. if they determine the same things. This seems reasonable, even if necessary coextensiveness is not a sufficient condition for property identity in general. For fusions of natures are uniquely decomposable: so any nature in A, say N_a, corresponds to a nature in B, say N_b. N_a and N_b determine the same thing, as they cannot differ in the qualitative features they bestow on it, so they are indiscernible and, being universals, identical.

Up to now, we have defined membership only for the special case of membership-in-a-singleton:

(4) $a \in \{b\} \quad :\Longleftrightarrow \quad a$ is determined by the nature of b

From $a \in \{b\}$, we would like to deduce $a = b$. Prima facie, this seems to founder on the fact that Identity of Indiscernibles is not a necessary truth with respect to particulars. Arguing this way, however, we overlook that Diversity of Distinguishables (1.2) is all we need, for $=$ is defined not as identity (in the strict or metaphysical sense), but as (qualitative) indiscernibility in the first-order predicate calculus in which we formulate the axioms of set theory.[28] We can now use (3) to generalize (4) in the following way:

(5) $a \in A \quad :\Longleftrightarrow \quad a \in \{a\} \wedge \{a\} \subset A \quad \Longleftrightarrow \quad a$ is determined by A

Whenever the nature of a is part of a fusion of natures (a class) A, A determines a. Lewis has argued that one class is part of another if the first is a subclass of the second.[29] "$\subset$", the subclass relation between classes, is just ordinary parthood on Lewis' account. On ours, it is property entailment restricted to (fusions of) natures. Property entailment was stipulated to be a partial order and so we can simply identify it with parthood.[30] So we do not differ from Lewis in this respect. Having natures at our disposal, we can let mereology go proxy for set-theory.

6 Sets as properties

I have argued that the nature of a is a good candidate for $\{a\}$. Whether or not I am right in claiming this, depends on whether such a construal of singletons makes sense of set theory in general. In this section, therefore, I want to reinterpret the familiar Zermelo-Fraenkel axioms of set-theory in property-theoretic terms and to show how they could be made plausible (or at least that they are not less plausible than their set-theoretical cousins).

We have already seen how to ban the spectre of two-membered singletons by taking the "=" of first-order predicate calculus for what it is, namely qualitative indiscernability. This renders the following wording of the axiom of extensionality:

[27] Note that we need to give the quantifier in (3) a possibilist reading for antisymmetry.

[28] Someone worried about transitivity should be reminded that we speak of indiscernability not with respect to our discriminatory capacities, but of indiscernability in principle, i.e. identity with respect to all properties. Even if we dislike axiom schemata and prefer a second-order axiomatization, the situation does not change, as identity is then definable as $\forall F(x = y \Longleftrightarrow (Fx \Leftrightarrow Fy)$.

[29] This is the less controversial half of Lewis' 'First Thesis'. It has been by Alex Oliver (1994) on the ground that Lewis' linguistic evidence seems inconclusive. As I am interested, however, in the application of mereology to sets, I will ignore Oliver's criticisms in the following.

[30] Chisholm (1982: 143) chooses "inclusion" for the relation between properties A and B which holds iff whatever exemplifies A exemplifies B. Someone wishing to deny our identification of $\Vdash$ with $\subset$ would have to give a *metaphysical* argument, for it can be proved that any partial order on sets is isomorphic to $\subset$.

6.1 (Extensionality Axiom) *If two things are indiscernible, they have the same nature. (If two sets have identical elements, then they are equal:* $\forall x \forall y \forall z(x = y \leftrightarrow (z \in x \leftrightarrow z \in y)))$

The nature of a, *having all and only the properties of* a, gives us a complete description of *one* thing. Frege noticed that we cannot but count by properties: independent of the metaphysical status of Identity of Indiscernibles, we do not have use for distinct indiscernibles in mathematics. If there were two particulars with the same nature, we would count them as one. Natures are thus 'unifying' in a way other properties are not. To see why (6.1) really is equivalent to (1.2), assume that two sets, i.e. fusions of natures, differ; then one of them has a nature as a part that the other does not. Because any nature is exemplified (by (1.3)), there will be a thing determined by one, but not by the other of the fusions. So they determine different things and thereby do not have identical elements.

(6.1) is *obviously* true. It even has a good claim to analyticity, if anything has: (6.1) follows from our definition of nature as that which accounts for all qualitative features of *one* thing. Boolos has said the same of the extensionality axiom of set theory: although it does not follow from the iterative conception, it is already implied by what we mean by "set". If sets were not extensional, they would not be sets.[31] On our account, (6.1) follows from the specificity of natures, which allows for fusion of natures to be uniquely decomposed into parts that are natures, a principle which mimics the unique decomposition into simple parts familiar in standard extensional mereology.

We now turn to two existence claims, the first of which we already discussed above:

6.2 (Null Set Axiom) *There is a property which is not the nature of anything. (There is a set which has no members:* $\exists x \forall y (y \notin x))$

6.3 (Axiom of Infinity) *There is a fusion of natures which determines* $\emptyset$ *and the nature of everything it determines. (There is a set* x *containing* $\emptyset$ *and closed under the successor operation:* $\exists x \forall y (\emptyset \in x \wedge (y \in x \rightarrow \{y\} \cup y \in x)$

The successor operation is the function S giving us the von Neumann ordinals: $S(\alpha) := \alpha \cup \{\alpha\}$. It is definable in terms of singleton formation, which we get from the Pairing Axiom: $\forall x \exists y \forall z (z \in y \leftrightarrow z = x)$. What (6.3) claims, then, is that there is something having an infinitely complex nature, something which is rich enough to determine something and its own ressources used in its determinations. Transitive sets in general, and ordinals in particular, correspond to such recursively self-determinating properties. Ontologically, (6.3) gives us an infinite chain of smaller and smaller parts of natures: a babushka-like entity which is contained in the one urelement we started with. A further classical, though redundant,[32] axiom is the following:

6.4 (Pairing) *Any two things are determined by the fusion of their natures. (For any two things there is a doubleton:* $\forall x, y \exists z \forall u (u \in z \leftrightarrow u = x \vee u = y))$

Next, we have to consider the two operations giving us the cumulative hierarchy.

[31] Boolos gives a mereological explanation for this: "If so [if the set is nothing other than its members], then extensionality follows from the transitivity of identity: for if every member of x is a member of y and vice versa, then the members of x *are* the members of y; therefore x, i.e., the members of x, is identical with y, i.e., the members of y, and extensionality holds." (1989: 93)

[32] We get the Pairing Axiom by applying the Axiom of Replacement to $\mathcal{P}(\emptyset), \mathcal{PP}(\emptyset), \mathcal{PPP}(\emptyset), \ldots$

6.5 (Powerset Axiom) *If x is a fusion of natures, there is a fusion of natures determining anything which determines some of the things determined by x. (If x is a set, there is a set that consists of all and only the subsets of x: $\forall x \exists y \forall z (z \subset x \rightarrow z \in y)$)*

6.6 (Axiom of Union) *If x is a fusion of natures, there is a fusion of natures determining exactly what is determined by what is determined by x. (If x is a set, there is a set whose members are precisely the members of the members of x: $\forall x \exists y \forall z (z \in y \leftrightarrow \exists u \in x (z \in u))$)*

The Axiom of Union is unproblematic: it says that whenever something is mediately determined by a fusion of natures, there is another fusion by which the determination is immediate. The Powerset Axiom is much stronger. It says that any subset of x, i.e. anything determining some, but not necessarily all of the things determined by x, is itself determined by another fusion of natures. This means that fusions of natures are transparent in the sense that their existence implies that there is something determining all their parts (and nothing else). What these parts have in common, by (2), is that they determine some of the things determined by their whole. (6.5) means that this property is 'thick' enough to enable us to account for all the qualitative features bestowed by fusions of natures having this property.

The next two axiom schemata fix the link between language and ontology:

6.7 (Axiom of Replacement) *If we have a rule for determining new instances from old ones, we get the fusion of the natures of the new ones if we already have the fusion of natures of the old ones. (Let $\phi(x, y)$ be any formula such that for each set a there is a unique set b such that $\phi(a, b)$. Let x be a set. Then there is a set y consisting of just those b such that $\phi(a, b)$ for some a in x: $\forall x((\forall a \in x)(\exists! b \in x)\phi(a, b) \rightarrow \exists y(\forall z \in x)(\exists b \in y)\phi(z, b))$)*[33]

6.8 (Axiom of Separation) *Whenever some things determined by a fusion of natures share a property F, there is a fusion of natures that determines just these things and no others. (Every set has a subset of all and only its elements that are Ψ: $\forall x \exists y \forall z((z \in y) \leftrightarrow (z \in x \wedge \Psi(z)))$)*

If we have a total function defined on everything determined by some fusion of natures, we know how to determine its values, so there is something that determines the range of our function. What (6.7) says, then, is that determination is preserved under functional dependencies. The limited comprehension says that determination is preserved under the inclusion of further properties: (6.8) assures us that whenever something determines a number of things, there is something that determines just those of these things that share a further property. The fusion of natures that determines these things consists (is the 'subfusion') of those natures that include the further property. If we compare (6.8) with the unrestricted comprehension principle, we see why the former had better be valid and the latter not.

6.9 (Unrestricted Comprehension) $\exists y \forall x((x \in y) \leftrightarrow \Psi(x))$

The problem with unrestricted comprehension is that nothing assures us that it is possible to determine all and only the Ψs in the first place. For any given number of things, to be sure, there is a property shared by all and only these things, namely the property of being determined by the fusion of their natures. This property, however, does not always amount to a nature. If we had unrestricted comprehension, it would

[33] As I would like to stay neutral on the question whether there are any non-wellfounded sets, I actually need something stronger, i.e. a collection axiom where the unique-existential quantifier is replaced by a normal, non-unique existential quantifier.

have to be possible that this property is the *only* one shared by a given number of things (except all the properties entailed by it and *existence*) and this might well be impossible in pathological cases.

There are properties that are had by themselves, e.g. *existence*. It might seem, however, that no such property can be a nature, for it then would have to include itself (via it's totality property) as a proper part. This is stated by the next axiom:

6.10 (Axiom of Foundation) *Nothing is its own nature. ($\in$ is a well-founded relation; $\forall x(x \neq \emptyset \rightarrow \exists y \in x \forall z \in x(z \notin y))$)*

The requirement that there be no infinitely descending or circular $\in$-chains means that determination comes to an end somewhere. The last axiom to be discussed is the Axiom of Choice:

6.11 (Axiom of Choice) *If some fusions of natures are such that nothing is determined by more than one of them, then there is a fusion of natures determining exactly one of the things determined by them. (For every set x of pairwise-disjoint nonempty sets there is a choice set, i.e. a set that consists of precisely one element from each member of the first set: $\exists z \forall y \in x(\mid z \cap y \mid = 1)$)*

The intuitive idea behind (6.11) is this: For any thing x determined by the original fusion of natures, take the (unique) fusion of natures that determines x and the nature of x as its part, fuse these natures together and you have the choice set you were looking for. Involving reference in this way to something like a human activity, we can understand why (6.11) is so obvious in the finite and so powerful in the infinite case. The main idea is that you do not only get the members, but also their natures, out of a fusion of natures: Such fusions are transparent not only with respect to what they determine, but also with respect to what they have to contain in order to determine them. This transparency suggests the presence of an order permitting a choice and (6.11) really is equivalent to Zermelo's Well-Ordering principle: $\forall x \exists r(r$ is a wellorder of $a)$.

7 Paradoxes and Proper Classes

Sets are those classes that are members; so they are those fusions of natures that themselves are uniquely determined by their natures. Some classes, however, are not members of anything; so, in particular, they do not have a nature. Given my claim that anything whatsoever, including bare particulars, has a nature, this may seem problematic. I therefore have to qualify my claim: Nothing can fail to have a nature for *lack* of properties; it may well be, however, that some things cannot be determined uniquely for *abundance* of properties. They are such that anything determining them has to be so inclusive that it also determines something else.

Some predicates of the form "having $F, G, \ldots$ as only properties" do not denote any property. Any nature of which their referents would be a part of are unexemplified and hence, by (1.3), do not exist. Some combinations of properties cannot be all and only the properties of something; not everything that seems to be a totality *property* is one.

I am thus following Lewis in not blaming mereology for the set-theoretic paradoxes, but the making of singletons (1991: 19). But whereas Lewis' accusation has stroken some as unjust,[34] It seems to me that my account explains why the step from

[34]So it did to Potter: "But to blame the singleton operator for Russell's paradox in this fashion [of Lewis'] is surely a case of shooting an innocent bystander, since the consistency of the singleton operator *on its own* is not in question." (1993: 364) The claim is not, however, that the singleton operator is somehow inconsistent, but just that it is not everywhere defined.

things to their nature can end us in paradox: the set-theoretic paradoxes are really property-theoretical in nature.[35]

The class of all non-self-members is the fusion of the natures of all those things that do not exemplify themselves. The nature of this class does not exist, by the familiar argument.[36] Anything that determines all the non-self-members would have to be so inclusive as to determine also something else, e.g. itself.

Another paradox 'mirrored' in what seems to be a natural way in our theory is Cantor's. Suppose there is a property determining everything. This property, by (1.3), would have to be exemplified. So it would be exemplified by everything and, like *existence*, be exemplified by itself and thus lack a nature.[37]

8 Why singletons are not states of affairs

Armstrong's view is that singletons are mereologically atomic, though not atomic *tout court*. They do not have parts, but they have what he calls *constituents*. Their constituents are things and properties; their mode of composition is the one building states of affairs out of particulars and universals. $\{a\}$ is an entity of the sort *a's being* F, where F is a "unit-determining" property, i.e. a property such that, necessarily, "a thing falling under [it] is just *one* thing of that sort, one to the exclusion of other numbers" (1991: 197). To secure that such a property exists for any thing that has a singleton, Armstrong in (1989: 134) chooses the disjunction of all unit-determining properties of a (not itself a universal but supervenient upon its disjuncts) and in (1991: 197) the second-order property *having some unit-determining property*. To secure the existence of unit-determining properties, Armstrong then jettisons his former taste for desert landscapes and countenances "any true relational description of an entity as yielding a property of an object, and a state of affairs in which that object figures" (1991: 198) and thus as constituting a singleton. While this certainly secures enough singletons, we might suspect that it gives us too much. And indeed it is difficult to see how Armstrong could allow for proper classes.[38]

In (1989: 137), Armstrong identified the empty set with a negative existential state of affairs, namely the disjunction of all possible but (actually) unexemplified properties failing to be true of anything whatsoever. In (1991: 199) he took it to be a negative state of affairs which supervenes on the "all-embracing totality state of

[35] This has been obscured by a familiar way of telling the story of Russell's paradox, according to which the latter shows that not every *property* defines a set. This account of the story has been criticised persuasively by John Myhill (1984), who quotes Gödel to the effect that "there never were any set-theoretic paradoxes, but the *property-theoretic* paradoxes are still unresolved." (1984: 129).

[36] Assume it exists. Then it is either exemplifies itself or it does not. Exemplifying itself is being part of one's nature. If the class of all non-self members is part of its nature, its nature is part of the fusion of the natures of those things that do not exemplify themselves. So it does not exemplify itself and thus is not part of its nature. If it is not part of its nature, on the other hand, its nature is part of the fusion, so is part of it, so it does exemplify itself after all.

[37] A similar difficulty occurs with Armstrong's totality facts. He claims that the regress is harmless: once the collection of all states of affairs and the first totality fact (that this collection contains all the states of affairs there are) are in place, further totality facts are supervenient on those and "involve no increase of being", because they are entailed by the first totality fact (1997: 198). This seems false, however: If supervenient entities are nothing over and above their supervenience base, they cannot depend for their existence on more things than their base. But if "This [followed by a very long list] are all and the only states of affairs there are" has a truthmaker, this must be another state of affairs appearing in the list. So it would be a Liar-like sentence, claiming that its truthmaker obtains and hence that it is true. So it cannot, on itself, assure us of its own truth, but requires something else to to that.

[38] In (1989: 135), he seems to adopt an unrestricted comprehension principle: "All that is required, I suggest, is that some *description* be contingently true of the set member. [...] It seems reasonable that this demand for a contingent description can always be met." Cf. also (1997: 185).

affairs: that a certain collection of entities is all that there is". It does not seem clear, however, how this totality state of affairs could fail to be the singleton of everything there is. It certainly ascribes a unit-making property to the collection of everything. Does it then include itself or not?

There are three further problems with Armstrong's approach: The first one is his appeal to non-mereological composition, of which we do not know much more than that it is not unrestricted, not functional and not ontologically innocent. The second one is that the ontological price seems too high. The third and most general reason for being critical of Armstrong's proposal is that it seems doubtful whether states of affairs exist. Though I cannot go into these matters here, an ontologically less committal proposal seems preferable.

In a state of affairs, a particular and a universal are bound up with one another in a non-mereological way. States of affairs are what makes propositions true. So it seems that a's having *some* unit-determining property obtains because another still more basic state of affairs obtains, namely a's having a specific unit-determining property F. Armstrong cashes this out in terms of supervenience and his doctrine of the ontological free lunch: $\{a\}$ is 'no increase in being' on Fa.[39] As the requirement for truthmakers is the principal reason to postulate states of affairs in the first place, the ontological status of a state of affairs not making anything true seems dubious.

Suppose *red* is an universal. So it has a singleton, $\{red\}$, which has a singleton and so on. So there is an infinity of different, indeed mereologically disjoint, states of affairs having *red* for one of its constituents and ever more refined relational properties of *other* states of affairs involving *red* for the other.[40] This seems a rather extravagant ontology indeed. Moreover, it is not enough that "under these relaxed rules, it would *seem* that any member of any singleton has a monadic states of affairs associated with it" (1991: 198, my emphasis) for this can *be proved* in standard set theory. It seems weird to let mathematics decide on the existence of an infinity of different states of affairs involving one ordinary universal.

By making singletons states of affairs, Armstrong reverses the modal asymmetry we observed earlier: given his Factualism (the claim that particulars are mere abstractions from the ontologically more basic states of affairs), it would seem that a depends on and is somehow derived from $\{a\}$.

[39] Not only the mode of composition forming states of affairs is non-mereological, but also the way properties are put together to form conjunctive states of affairs (1997: 36). It is therefore not clear how Armstrong can maintain the thesis that plural sets, which he construes as conjunctive states of affairs (1989: 134), are fusions of their singletons. For all we know of the property making up the state of affairs is that it is complex; and to decompose the set into its members, we have to know that the conjunctive state of affair is of the form $Fa \wedge Gb$ rather than $Fa \wedge Ga$. To know this, however, "it may be necessary [...] to specify whether the particulars involved are identical, wholly distinct *or standing in part/whole or overlap relations.*" (1997: 37)

[40] Armstrong is right that $\{a\}$ supervenes on some basic state of affairs involving a (1991: 199), i.e. the state of affairs of a's having the unit-determining property which makes that *having a unit-determining property* is true of a and hence that $\{a\}$ exists. For $\{\{a\}\}$ to exist, however, there has to be *another* state of affairs involving not a but $\{a\}$. So the regress is not ontologically innocent, but generates new states of affairs at any level and indeed bizarre ones of the form "the state of affairs['] of a certain state of affairs obtaining" (1989: 136). Moreover, if $\{red\}$ is actual, so will be these states of affairs, thus rendering Armstrong's claim that the universe might consist (and probably consists) of only finitely many things provably false. It does not help, as Armstrong seems to things (1997: 193), to require only the *possibility* of a unit-determining property for mere possibilities require actual truthmakers or at least actual universals. It seems doubtful (and especially so for actualists like Armstrong) that something that would be a unit-determining property of the universe and which can be *proved* not to exist might still be exemplified by something other, something that is not everything there is. The withdrawal to possibility does not help with the paradoxes either. The paradox that Armstrong derives from the set of all singletons (1997: 195) just turns into the paradox of the set of all *possible* singletons. Moreover, it seems dubious what the empty set would be on a possibilist account.

9 Why singletons are not haecceities

John Bigelow, in his (1989), has identified singletons with what he calls "individual essences": $\{a\}$ is a universal a must have and which it cannot share with other things. $\{a\}$ and a are mutually dependent: each cannot exist without the other (1989: 295). Bigelow backs this up by an appeal to extensionality: to be a member of $\{a\}$, something has to be a, and nothing, conversely, cannot be a member of $\{a\}$ without being a (1989: 296, 1993: 82). Here, however, lurks a confusion: *being a member of* $\{a\}$ is not the same thing as $\{a\}$; the former, but not the latter is a *relational* property of a.[41] Bigelow thus presupposes rather than shows that sets are properties of their members. Another problem concerns the allegedly mutual dependence of singletons and their members. If a singleton $\{a\}$ essentially has the member it has and cannot share *having only a as its member* with other things, then it seems that *having only a as its member* is the (or a) individual essence of the singleton, i.e. it's singleton. $\{\{a\}\}$, however, does in the same way mutually depend on a, so it also seems to be the (or a) individual essence of it. So we get a circle instead of a hierarchy.

Sets in general are 'plural versions of individual essences' (1989: 295), 'plural essences' (1993: 83) or 'pluralized individual' essences (1993: 84), i.e. essential properties of each of their members. They are thus properties shared by their members. The problem with this is that it gets the part/whole-relation wrong.[42] Part/whole-relations among universals, according to Bigelow (1988: ??, 1989: 297), ground entailment relations: it is because F is a part of $F\&G$ that nothing can be both F and G without being F. With sets, however, entailment is the other way round: if $x \subset y$, every member of x is a member of y, i.e. membership in y entails membership in x.[43] In Bigelow's framework, this means that any sharable property entails the individual essences of the objects sharing it among them. If something exemplifies the plural essence $\{a, b\}$, say by having $\{b\}$ as its individual essence, it does not follow that it also exemplifies $\{a\}$. Another problem concerns the contrast to other properties like *being hungry* the exemplification of which does not, unlike that of sets, entails being identical to one of actual things exemplifying it. Because a person "could be hungry without having to be numerically identical with one of the things that happen in the actual world to constitute all and only the hungry" (1993: 84), such a person could be hungry without being a member of the set of all and only the hungry persons.

Another, more important, problem is that it is not clear why every thing has one, and only one, individual essence. Recognizing this (1993: 85), Bigelow returns to a proposal he rejected in (1989: 299) and tentatively construes $\{a\}$ as the *haecceity* of a, i.e. the unstructured, primitive 'thisness' of a (1993: 86). The problem with this is the following: Even granted that there are haecceities, there simply are not enough of them to go proxy as singletons. Even if particulars and universals have haecceities, haecceities do not. For a haecceity F already is what makes something unique and if it itself would need a haecceity to be unique it would not do what it has been introduced for in the first place. A second-order haecceity is a contradiction in terms.

[41] Bigelow seems to think that this difficulty can be overcome by recognizing that membership is exemplification: "Thus, when we say that 'being a member of $\{a\}$' is an essential property of a, this is a roundabout way of saying something crisper: that the unit set $\{a\}$ is itself an essential property of a." (1993: 83) Instead, it seems to me an equally crisp way to say something *different*.

[42] Another problem pertains already to the formulation of the claim that sets "could not have had any other membership than that which they do have" (Bigelow/Pargetter 1990: 368). In their book, Bigelow and Pargetter formulate this principle as "$\forall y(\Box(y \in x) \vee \Box\neg(y \in x))$", thereby presupposing that it makes sense to talk of sets existing 'in' a world.

[43] Bigelow denies this (1989: 298, line 12), though the mistake is so evident that one might suspect a printer's error.

The last feature of Bigelow's theory I would like to discuss pertains to the empty set. Bigelow himself admits that it is a problem: "It is not easy to see how the empty set could plausibly be construed as either a singular or a plural essence." (Bigelow/Pargetter 1990: 374). To explain what the empty set is, Bigelow makes appeal to another, the so-called 'higher-order' theory of sets, according to which sets are what the instances of coextensive universals have in common. What disjoint universals have in common, however, is not exemplified by their instances, but by the universals themselves. So while non-empty sets are second-order universals, i.e. properties of their members, the empty set is not.

10 Concluding remarks

I hope to have made clear why I think that singletons could well be natures and why philosophers accepting particulars, universals and sets could spare themselves one category. I have not tried, however, argued against structuralism and I have not dealt with the problem on what basis we could decide between different, though mathematically isomorphic and metaphysically equally plausible proposals as to what singletons are. For the present, I am satisfied with having sketched at least *one* plausible account of what singletons could be.