

# Analytic vs. Synthetic Methods in Metaphysics

Philipp Blum

April 20, 2017

There are many reasons to be skeptical about Descartes' argumentative project in the first part of the *Meditations* – to conclude substance dualism from the fact that even maximal doubt cannot encompass the doubter's existence. One particular, quite general one, is the following: how could such a radical metaphysical premiss *ever* follow from merely epistemological premisses, having to do with what a thinker, even a properly idealised one, can and cannot rationally do?

More concretely and assuming that Descartes' overall argument has roughly the structure of a *reductio*: what are the conditions under which, having made a supposition *s*, and having shown that under this supposition, *s* could not have been supposed, we are entitled to conclude that *s* is false? Do we not have to make the additional supposition not only that *s*, but also that we are supposing *s*, and if so, how could we ever know *that*?

When faced with (something like) this objection, Descartes emphasises that he is using, in metaphysics and also quite generally, “the analytic method”. He never quite explicitly says what the analytic method is, but he is emphatic that he is following it and that this fact is important for addressing certain objections:

“Et il est à remarquer, en tout ce que j'écris, que je ne suis pas l'ordre des matieres, mais seulement celui des raisons: c'est à dire que je n'entreprends point de dire en un mesme lieu tout ce qui appartient à une matiere, à cause qu'il me seroit impossible de le bien prouver, y ayant des raisons qui doivent estre tirées de bien plus loin des unes que les autres; mais en raisonnant par ordre à *facilioribus ad difficiliora*, j'en déduis ce que je puis, tantost pour une matiere, tantost pour une autre; ce qui est, à mon avis, le vray chemin pour bien trouver&expliquer la verité.” (AT III 266<sup>16–26</sup>)

“It should be noted that throughout the work the order I follow is not the order of subject-matter, but the order of reasoning. This means that I do not attempt to say in a single place everything relevant to a given subject, because it would be impossible for me to provide proper proofs, since my supporting reasons would have to be drawn in some cases from considerably more distant sources than in others. Instead, I reason in an orderly way ‘from what is easier to what is harder’, making what deductions I can, now on one subject, now on another. This is the right way, in my opinion, to find and explain the truth.” (? : 163)

For Descartes, analysis is the right method not only in philosophy, but also in mathematics (AT II 637<sup>12–17, 24–27</sup>): it not only presents results already found, but proceeds “a *facilioribus ad difficiliora*”, thereby showing *how* the principles have been found. He explicitly says that his *Meditations* show the analytic method at work.<sup>1</sup>

But what *is* the analytic method? The most prominent explanation of the distinction between the analytic and the synthetic method is in the *Second Replies*, where it is a sub-distinction within the ‘method of the geometers’.

Analysis veram viam ostendit per quam res methodice & tanquam a priori inventa est, adeo ut, si lector illam sequi velit atque ad omnia satis attendere, rem non minus perfecte intelliget suamque reddet, quam si ipsemet illam invenisset.

L'analyse montre la vraye voye par laquelle une chose a esté methodiquement inventée, & fait voir comment les effets dépendent des causes; en sorte que, si le lecteur la veut suivre, & jeter les yeux soigneusement sur tout ce qu'elle contient, il n'entendra pas moins parfaitement la chose ainsi démontrée, & ne la rendra pas moins sienne, que si luy-mesme l'avait inventée.

Nihil autem habet, quo lectorem minus attentum aut repugnantem ad credendum impellat; nam si vel minimum quid ex iis quae proponit non advertatur, ejus conclusionum necessitas non apparet, saepeque multa vix attingit, quia satis attendenti perspicua sunt, quae tamen praecipue sunt advertenda. (AT VII 155<sup>23–156<sup>3</sup></sup>)

Mais cette sorte de demonstration n'est pas propre à convaincre les lecteurs opiniastres ou peu attentifs: car si on laisse échapper, sans y prendre garde, la moindre des choses qu'elle propose, la nécessité de ses conclusions ne paroitra point; & on n'a pas coûtume d'y exprimer fort amplement les choses qui sont assez claires de soy-mesme, bien que ce soit ordinairement celles ausquelles il faut le plus prendre garde. (AT IX/I 121)

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1. “Ego vero solam Analysim, quae vera & optima via est ad docendum, in Meditationibus meis sum sequuntus...” (AT VII 156<sup>21–23</sup>)

Synthesis è contra per viam oppositam & tanquam a posteriori quæsitam (etsi sæpe ipsa probatio fit in hac magis a priori quam in illâ) clare quidem id quod conclusum est demonstrat, utiturque longâ definitionum, petitionum, axiomatum, theorematum, & problematum serie, ut si quid ipsi ex consequentibus negetur, id in antecedentibus contineri statim ostendat, sicque a lectorem quantumvis repugnante ac pertinaci, assensionem extorqueat; sed non ut altera [demonstrandî ratio] satisfacit, nec discere cupientum animos explet, quia modum quo res fuit inventa non docet. (AT VII 156<sup>6-16</sup>)

La synthese, au contraire, par une voye toute autre, & comme en examinant les causes par leurs effets (bien que la preuve qu'elle contient soit souvent aussi des effets par les causes), démontre à la verité clairement ce qui est contenu en ses conclusions, & se sert d'une longue suite de definitions, de demandes, d'axiomes, de theoremes & de problemes, afin que, si on luy nie quelques consequences, elle face voir comment elles sont contenuës dans les antecedens, qu'elle arrache le consentement du lecteur, tant obstiné & opiniastre qu'il puisse estre; mais elle ne donne pas, comme l'autre [manière de démontrer], une entiere satisfaction aux esprits de ceux qui desirent d'apprendre, parce qu'elle n'enseigne pas la methode par laquelle la chose a esté inventée. (AT IX/I 122)

Analysis is here said to be the method of invention, leading to what is clear by itself, while synthesis is the method of demonstration and exposition, dialectically useful but not ultimately explanatory. The French translation, authorised by Descartes, makes clear that analysis is “a priori” in the sense of going from effects to causes, while synthesis is “a posteriori” in the sense of ‘deducing’ ‘effects’ from their causes. It thus aligns the analytic/synthetic dichotomy with the one used in the so-called ‘Padua school’, distinguishing on the basis of the *Analytica Posteriora* the method of resolution, which goes from what is prior to us to what is prior in itself, from the method of composition, which proceeds in the other direction.

In *Analyt. Post.*, Aristotle draws the distinction as one between understanding *that* and understanding *why*:

Understanding the fact and the reason why differ, first in the same science – and in two ways. [...] In a second way if, although the deduction does proceed through immediates, it proceeds not through the explanation but through the more familiar of the converting terms. For there is no reason why the non-explanatory counterpredicated term should not sometimes be more familiar, so that the demonstration will proceed through this term. (*APst* I 13 78a22ff.)

We are given the following example:

The planets don't twinkle. <u>What does not twinkle, is near.</u> The planets are near.	The planets are near. <u>What is near, does not twinkle.</u> The planets don't twinkle.
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The syllogism on the left side infers the cause from the effect, and uses the ‘more familiar’ second premiss, inferring not in virtue of “the explanation” (the nearness of the planets), but in virtue of the “more familiar” of the terms (the non-twinkling). On the right hand-side, by contrast, the effect is demonstrated from the cause, and we are given the reason why the planets appear to us the way they do.

Synoptically, we thus get the following picture:

<i>Analysis</i> resolutio demonstratio quia syllogismos tou hoti proof of the cause from the effect explanation of the fact rationes cognoscendi from the conditioned to the condition from first <sub>1</sub> to first <sub>2</sub> ”démontrer” in the sense of “explain” ”fait voir comment les effets dépendent des causes”	<i>Synthesis</i> compositio demonstratio propter quid syllogismos tou dioti proof of the effect from the cause explanation of the reason why rationes essendi from the conditions to the conditioned from first <sub>2</sub> to first <sub>1</sub> ”démontrer” in the sense of “deduce” ”comme en examinant les causes par leurs effets”
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## Mathematical methods

The dichotomy appears in the thirteenth book of Euclid's *Elements*:

Analysis is the assumption of that which is sought as if it were admitted <and the arrival> by means of its consequences at something admitted to be true. Synthesis is an assumption of that which is admitted <and the arrival> by means of its consequences at something admitted to be true. (? : 442)<sup>2</sup>

Proclus, in his commentary on the *Elements* explicitly says that the methods only differ in direction:

...certainly beauty and order are common to all branches of mathematics, as are the method of proceeding from things better known to things we seek to know and the reverse path from the latter to the former, the methods called analysis and synthesis. (translation after ? : 6–7)

Pappus of Alexandria, in his *Collectio* explicates the distinction as follows (citing the Commandinus translation):

”Resolutio igitur est via a quaesito tamquam concesso per ea, quae deinceps consequuntur [*δια τὴν ἑξῆς ἀκολούθῳ*] ad aliquod concessum in compositione: in resolutione enim id quod queritur tamquam factu ponentes, quid ex hoc contingat [*το ex ὁυ τούτο συμβαίνει*], consideramus: & rursus illius antecedens, quousque ita progredientes incidamus in aliquod iam cognitum, vel quod fit è numero principiorum. [...] In compositione autem per conversionem ponentes tamquam iam factum id, quod postremum, in resolutione sumpsimus: atque hic ordinantes secundu naturam ea antecedentia, quae illic consequentia erant & mutua illosum facta compositione ad quasiti finem pervenimus, & hic modus vocatur compositio.” (?: 328–329)

Now, analysis is the path from what one is seeking, as if it were established, by way of its consequences, to something that is established by synthesis. That is to say, in analysis we assume what is sought as if it has been achieved, and look for the thing from which it follows, and again what comes before that, until by regressing in this way we come upon some one of the things that are already known, or that occupy the rank of a first principle. [...] In synthesis, by reversal, we assume what was obtained last in the analysis to have achieved already, and, setting now in natural order, as precedents, what before were following, and fitting them to each other, we attain the end of the construction of what was sought. This is what we call ‘synthesis’. (? : 93)

While the reversability, turning the analytic discovery into a synthetic proof, is key to its use in mathematics (and favours, in my view, an interpretation according to which the methods deal with the transformation of equations), it is potentially problematic, because circular, in its use in science and metaphysics. Descartes replies to this charge in a letter (22nd of February 1638) to Morin:

Vous dites aussi que *prouver des effets par une cause, puis prouver cette cause par les memes effets, est un cercle logique*, ce que j’avoüe; mais je n’avoüe pas pour cela que c’en soit un [cercle logique], d’expliquer les effets par une cause, puis de la prouver par eux: car il y a grande difference entre *prouver* & *expliquer*. A quoy j’adioute qu’on peut user du mot *demonstrer* pour signifier l’un & l’autre.... (AT II 197<sup>25</sup>–198<sup>6</sup>)

You say also that there is a vicious circle in proving effects from a cause and the proving the cause by the same effects. I agree: but I do not agree that it is circular to explain effects by a cause, and then prove the cause by the effects; because there is a big difference between *proving* and *explaining*. I should add that the word ‘demonstrate’ can be used to signify either... (? : 106)

Here, the explanatory potential of the analytic method is stressed, which is also the reason why it is useful in science.

## Scientific methods

In the *Discours*, Descartes says about the method the fruits of which are shown in the *Essais*:

...de conduire par ordre mes pensées, en commençant par les obiets les plus simples & les plus aisez a connoistre, pour monter peu a peu, comme par degrez, iusques a la connoissance des plus composez; et supposant mesme de l’ordre entre ceux qui ne se precedent point naturellement les uns les autres. (AT VI 18<sup>27</sup>–19<sup>2</sup>)

...to direct my thoughts in an orderly manner, by beginning with the simplest and most easily known objects in order to ascend little by little, step by step, to knowledge of the most complex, and by supposing some order even among objects that have no natural order of precedence. (? : 120)

In practice, this means that he attempts to show the explanatory potential of first principles by sketching an abstracting model that shows them at work:

2. I am quoting Heath’s translation even though he thinks that “there must apparently be some corruption in the text.

Mais l'ordre que j'ay tenu en cecy a esté tel. Premierement, j'ay taché de trouver en general les Principes, ou Premières Causes, de tout ce qui est, ou qui peut estre, dans le monde, sans rien considerer, pour cet effect, que Dieu seul, qui l'a créé, ny les tirer d'ailleurs que de certaines semences de Veritez qui sont naturellement en nos ames. Après cela, j'ay examiné quels estoient les premiers & plus ordinaires effets qu'on pouvoit deduire de ces causes: et il me semble que, par la, j'ay trouvé des Cieux, des Astres, une Terre, & mesme, sur la terre, de l'Eau, de l'Air, du Feu, des Mineraux, & quelques autres telles choses, qui sont les plus communes de toutes & les plus simples, & par consequent les plus aysées a connoistre. Puis, lorsque j'ay voulu descendre a celles qui estoient plus particulieres, il s'en est tant presenté a moy de diverses, que je n'ay pas creu qu'il fust possible a l'esprit humain de distinguer les Formes ou Espèces de cors qui sont sur la terre, d'une infinité d'autres qui pourroient y estre, si c'eust esté le vouloir de Dieu de les y mettre, ny, par consequent, de les rapporter à nostre usage, si ce n'est qu'on vienne au devant des causes par les effets, & qu'on se serve de plusieurs experiences particulieres. En suite de quoy, repassant mon esprit sur tous les objets qui s'estoient jamais presentés a mes sens, j'ose bien dire que je n'y ay remarqué aucune chose qui je ne puisse assez commodement expliquer par les Principes que j'avois trouvez. Mais il faut aussy que j'avoué, que la puissance de la Nature est si ample & si vaste, & que ces Principes sont si simples & si generaux, que je ne remarque quasi aucun effect particulier, que d'abord je ne connoisse qu'il peut en estre deduit en plusieurs diveres façons, & que ma plus grande difficulté est d'ordinaire de trouver en laquelle de ces façons il en depend. (AT VI 63<sup>30</sup>–65<sup>3</sup>)

But the order I have adopted in this regard is the following. First, I tried to discover in general the principles or first causes of everything that exists or can exist in the world. To this end I considered nothing but God alone, who created the world; and I derived these principles only from certain seeds of truth which are naturally in our souls. Next I examined the first and most ordinary effects deducible from these causes. In this way, it seems to me, I discovered the heavens, the stars, and an earth; and, on the earth, water, air, fire, minerals, and other such things which, being the most common of all and the simples, are consequently the easiest to know. Then, when I sought to descend to more particular things, I encountered such a variety that I did not think the human mind could possibly distinguish the forms or species of bodies that are on the earth from an infinity of others that might be there if it had been God's will to put them there. Consequently, I thought the only way of making these bodies useful to us was to progress to the causes by way of the effects and to make use of many special observations. And now, reviewing in my mind all the objects that have ever been present to my senses, I venture to say that I have never noticed anything in them which I could not explain quite easily by the principles I had discovered. But I must also admit that the power of nature is so ample and so vast, and these principles so simple and so general, that I notice hardly any particular effect of which I do not know at once that it can be deduced from the principles in many different ways; and my greatest difficulty is usually to discover in which of these ways it depends on them. (? : 143–144)

Just before showing us these fruits, he says that in the synthetic presentation, the order of invention is reversed:

Car il me semble que les raisons s'y [in der *Dioptrique* und den *Meteores*] entresuivent en telle sorte que, comme les dernières sont démontrées par les premières, qui sont leurs causes, ces premières le sont reciproquement par les dernières, qui sont leurs effets. Et on ne doit pas imaginer que ie commette en cecy la faute que les Logiciens nomment un cercle; car l'experience rendant la plus part de ces effets tres certains, les causes dont je les deduits ne servent pas tant a les prouver qu'a les expliquer; mais, tout au contraire, ce sont elles qui sont prouvées par eux. (AT VI 76<sup>11-22</sup>)

For I take my reasonings to be so closely interconnected that just as the last are proved by the first, which are their causes, so the first are proved by the last, which are their effects. It must not be supposed that I am here committing the fallacy that the logicians call 'arguing in a circle'. For as experience makes most of these effects quite certain, the causes from which I deduce them serve not so much to prove them as to explain them; indeed, quite to the contrary, it is the causes which are proved by the effects. (? : 150)

In the *Traité de Lumière*, he qualifies the epistemic irresistibility of the first principles by their being distinctly conceived:

...qu'outre les trois loix que j'ay expliquées, je n'en veux point supposer d'autres, que celles qui suivent infailliblement de ces veritez éternelles, sur qui les Mathematiciens ont accoûtumé d'appuyer leurs plus certaines & plus évidentes demonstrations: ces veritez, dis-je, suivant lesquelles Dieu mesme nous a enseigné qu'il avoit disposé toutes choses en nombre, en pois & en mesure; & dont la connaissance est si naturelle à nos ames, que nous ne sçaurions ne les pas juger infaillibles, lors que nous les concevons distinctement; ny douter que, si Dieu avoit créé plusieurs Mondes, elles ne fussent en tous aussi veritables qu'en celui-ci. De sorte que ceux sçauront suffisamment examiner les consequences de ces veritez & de nos regles, pourront connoistre les effets par leurs causes; & pour m'expliquer en termes de l'Ecole, pourront avoir des demonstrations à *Priori*, de tout ce qui peut estre produit en ce nouveau Monde. (AT XI 47<sup>9-28</sup>)

...apart from the three laws I have expounded, I do not wish to suppose any others but those which follow inevitably from the eternal truths on which mathematicians have usually based their most certain and most evident demonstrations – the truths, I say, according to which God himself has taught us that he has arranged all things in number, weight and measure. The knowledge of these truths is so natural to our souls that we cannot but judge them infallible when we conceive them distinctly, nor doubt that if God had created many worlds, they would be as true in each of them as in this one. Thus those who are able to examine sufficiently the consequences of these truths and of our rules will be able to recognize effects by their causes. To express myself in scholastic terms, they will [be] able to have a *priori* demonstrations of everything that can be produced in this new world. (? : 97)

## Of any help in metaphysics?

In the *Regulae*, the reduction of what is complex to what is simple was simply called “the method”.

Tota methodus consistit in ordine et dispositione eorum, ad quae mentis acies est convertenda, ut aliquam veritatem inveniamus. Atque hanc exacte servabimus, si propositiones involutas&obscuras ad simpliciores gradatim reducamus, et deinde ex omnium simplicissimarum intuitu ad aliarum omnium cognitionem per eosdem gradus ascendere tentemus. (AT X 379<sup>15-21</sup>) The whole method consists entirely in

the ordering and arranging of the objects on which we must concentrate our mind’s eye if we are to discover some truth. We shall be following this method exactly if we first reduce complicated and obscure propositions step by step to simple ones, and then, starting with the intuition of the simple ones at all, try to ascend through the same steps to a knowledge of all the rest. (? : 20)

In the *Recherche*, this is advertised as a main advantage of the right method:

Omnes enim veritates se invicem consequuntur, & mutuo inter se vinculo continentur, totum arcanum in eo tantum consistit, ut a primis & simplicissimis incipiamus, & deinde sensim, &, quasi per gradus usque ad remotissimas & maxime compositas progrediamur. (AT X 526<sup>29-527</sup>)

...all truths follow logically from one another, and are mutually interconnected. The whole secret is to begin with the first and simplest truths, and then to proceed gradually and as it were step by step to the most remote and most complex truths. (? : 419-420)

And he retrospectively criticises the *Discours* for not cashing out this advantage of the analytic method:

...la principale cause de son obscurité [of the proof of God’s existence in the *Discours*] vient de ce que ie n’ay osé m’étendre sur les raisons des sceptiques, ny dire toutes les choses qui sont nécessaires *ad abducendam mentem à sensibus*: car il n’est pas possible de bien connoître la certitude & l’evidence des raisons qui prouvent l’existence de Dieu selon ma façon, qu’en se souvenant distinctement de celles qui nous font remarquer de l’incertitude en toutes les connoissances que nous avons des choses materielles...(AT I 560<sup>13-21</sup>)...the principal reason for

its obscurity is that I did not dare to go into detail about the arguments of the sceptics, or say everything which is necessary ‘to withdraw the mind from the senses’. The certainty and evidence of my kind of argument for the existence of God cannot really be known without distinctly revalling the arguments which display the uncertainty of all our knowledge of material things...(?: 86)

In the *Meditations*, he tries to do better, introducing methodological doubt right from the start:

“Quin & illa etiam, de quibus dubitamus, utile erit habere pro falsis, ut tantò clariùs, quidnam certissimum & cognitu facillimum sit, inveniamus.” (AT VIII/I 5<sup>12-14</sup>)

“Il sera mesme fort utile que nous rejections comme fausses toutes celles [de nos croyances] où nous pourrions imaginer le moindre doute, afin que, si nous en découvrons quelques-uns qui, nonobstant cette precaution, nous semblent manifestement vraies, nous facions estat qu’elles sont aussi tres-certaines, & les plus aisées qu’il est possible de connoître.” (AT IX/II 25)

It is by invoking methodological doubt, the “rules of Logic”, that the *Cogito*, understood as the proof of my existence as a thinking thing, may by the same token be taken to show that the *res cogitans* is ‘easier to know’ than the *res extensa*, even though we all start off as sensualists and believe only in the testimony of our senses:

Cùm primum ex rationibus in his Meditationibus expositis mentem humanam realiter a corpore distingui & notioerem esse quàm corpus, & reliqua collegissem, cogebar quidem ad assensionem, quia nihil in ipsis non cohaerens, atque ex evidentibus principiis juxta Logicae regulas conclusum, advertēbam. (AT VII 440<sup>1-6</sup>)

Lorsque j’eus la première fois conclu, en suite des raisons qui sont contenues dans mes Méditations, que l’esprit humain est réellement distingué du corps, & qu’il est mesme plus aisé à connoître que luy, & plusieurs autres choses dont il est là traité, je me sentois à la vérité obligé d’y acquiescer, pource que je ne remarquois rien en elles qui ne fust bien suivy, & qui ne fust tiré de principes tres-evidens, suivant les regles de la Logique. (AT IX/I 238-239)

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