

Leibnizian Space and Incongruent Counterparts

Philipp Blum, Leibniz in Ligerz, June 30, 2018

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Consider a left and a right hand. They are not related by an Euclidean motion, so at least weakly discernible. In contrast to Max Black's spheres, the hands much more clearly differ in dispositional properties: one, but not the other will fit a certain glove, e.g. While they do not differ in 'internal' relations, i.e. relations that hold between their parts, they do differ with respect to relations between them, in the sense that there are, between two non-congruent hands, differences that can be made to emerge in bigger world scenarios.¹

Throughout his career, Kant repeatedly used incongruent counterparts to show that our representations of space and time are intuitional rather than conceptual², in effect relativising the intrinsic/extrinsic distinction to the epistemic capacity by which we access the property.³

Their apparent possibility may be either taken as an argument for a substantialist account of space-time or as showing the need to acknowledge handedness as an intrinsic irreducible properties of spatial objects.

In Kant's case, the two hands differ by phenomenal properties, but these properties are neither extrinsic, nor relational, nor really response-dependent, but "innerlich, so weit die Sinne lehren". The two hands differ as phenomena, but not as noumena, in the same way as do the two raindrops discussed in the *Ambipholy*. Kant's there says that Leibniz's principle of the identity of indiscernibles holds of "concepts of things as such" ("von Begriffen der Dinge überhaupt gilt", A 272 / B 328):

...wenn ich einen Tropfen Wasser als ein Ding an sich selbst nach allen seinen innern Bestimmungen kenne, so kann ich keinen derselben von dem anderen für verschieden gelten lassen, wenn der ganze Begriff desselben mit ihm einerlei ist. Ist er aber Erscheinung im Raume, so hat er seinen Ort nicht bloss im Verstande (unter Begriffen), sondern in der sinnlichen äusseren Anschauung (im Raume), und da sind die physischen Örter, in Ansehung der inneren Bestimmungen der Dinge, ganz gleichgültig...(A 272 / B 328)

Leibniz's mistake to take the indiscernibility of identicals to hold not just of noumena, but of phenomena as well, is put in terms of forgetting about what was abstracted from:

1. Two figures are *incongruent counterparts* iff they are congruent, i.e. related by an Euclidean transformation, but not related by a proper motion, i.e. cannot be brought to coincide in space.

2. He says in his inaugural dissertation that "between solid bodies which are perfectly similar and equal but incongruent [...] there is a difference, [...] in spite of the fact that, in respect of everything which may be expressed by means of characteristic marks intelligible to the mind through speech, they could be substituted for one another. It is, therefore, clear that in these cases the difference, namely, the incongruity, can only be apprehended by a certain pure intuition." (1992: 403; "in solidis perfecte similibus atque aequalibus, sed discongruentibus [...] sit diversitas, [...] quanquam per omnia, quae notis, menti per sermonem intelligibilibus, efferre licet, sibi substitui possint, patet: hic non nisi quadam intuitione pura diversitatem, nempe discongruentiam, notari posse."; A2 20, 1983; V, 58)

3. In the *Prolegomena* (A 59, Kant says that the distinction between the right-hand and the left-hand glove is intrinsic to the senses ("innerlich, so weit die Sinne lehren") but is only possible by the relations of them to the whole of space of which they are a part ("die innere Bestimmung eines jeden Raumes ist nur durch die Bestimmung des äusseren Verhältnisses zu dem ganzen Raume, davon jener ein Teil ist (dem Verhältnisse zum äusseren Sinne), d.i. der Teil ist nur durch das Ganze möglich").

Weil aber bei dem blossen Begriffe von irgend einem Dinge von manchen notwendigen Bedingungen einer Anschauung abstrahiert worden, so wird, durch eine sonderbare Übereilung, das, wovon abstrahiert wird, dafür genommen, dass es überall nicht anzutreffen sei, und dem Dinge nichts eingeräumt, als was in seinem Begriffe enthalten ist. (A 281 / B 337-338)

Kant says here that Leibniz, thinking to be able to sensibly talk about noumena, abstracted from the extrinsic (i.e. spatio-temporal) properties that may distinguish intrinsically identical phenomena, took them to be non-existent and thus inferred the identity of things from the identity of their concepts.

It seems that handedness is not a property something has in virtue of standing in spatial relations to other things (and neither can it depend on relations to its parts, for these are the same among things differing only in their handedness). Relationists, however, have to deny that for there is in their view no fact of the matter whether a single hand in a universe is a left or a right hand. They cannot, like substantialists, explain handedness as a relation to the space something occupies.

If the existence of incongruent counterparts (and hence the exemplification of handedness) is contingent, then a relationalist may respond to Kant's thought experiment that a solitary hand would not have any handedness.

This is the reply Ishiguro gives on Leibniz's behalf to Kant about incongruent counterparts:

“Leibniz would probably have said to such an assertion that, although it is perfectly clear that in a world in which there are both left-hand and right-hand gloves one can make the distinction between the two, in a universe which consists solely of a single glove, this is not the case.” (Ishiguro 1990: 115)

There are two problems with such a reply, however:

1. If we identify the solitary left hand with the solitary right hand, are we not then violating the supposedly metaphysical (and not just mathematical) principle that identicals added to identicals cannot make for a difference? In other words: where does the intrinsic difference between the left-left and the left-right worlds come from?
2. If we identify the solitary left hand with the solitary right hand, how do we explain the fact that only one of them fits a right glove?

On the way to an answer to the first question it is useful to distinguish, with Khamara (2006: 65), three versions of the PII:

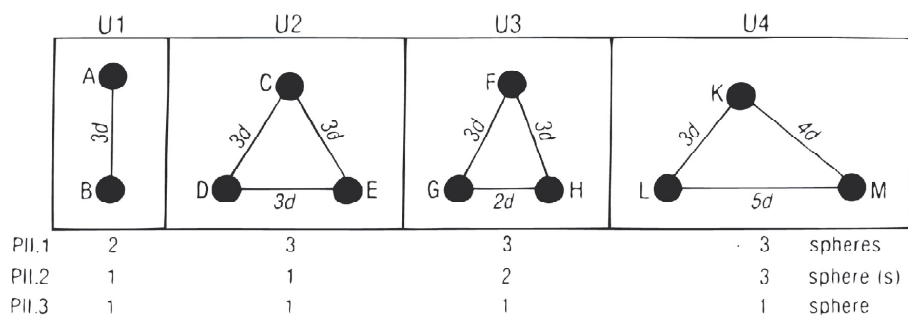
PII.1 no two things share all their properties

PII.2 no two things share all their pure properties (i.e.: properties the obtaining of which does not consist in the obtaining of a relation)

PII.3 no two things share all their intrinsic properties (i.e.: properties invariant under duplication)

With respect to four static universes consisting only of intrinsically identical spheres, the three principles yield the following verdicts (from Khamara (2006: 66)):

Static Universes



The verdicts of P11.1 and P11.3 are clear. For P11.2, the two spheres in U_3 are distinguished by the pure property of being 3 diameters away from some sphere. We get an odd result: “how can there be only 2 spheres, 3 diameters apart, if one of them has the property of being 2 diameters away from some sphere, which the other lacks” (Khamara 2006: 72)? Khamara’s diagnosis is apt:

“...what it shows is not that the 2-sphere verdict is incorrect, but that the correct verdict is incompatible with what the U_3 description [“consists of spheres F , G and H such that: G and H are each 2 diameters away from some sphere and 3 diameters away from some sphere; and F is 3 diameters away from some sphere and not 2 diameters ways from any sphere”] dictates when it is taken in isolation, namely that U_3 has 3 spheres. [...] Now such an inconsistency between the P11.2 verdict and the ‘pure’ description of a universe to which it is applied, is tantamount to declaring that universe logically impossible.” (Khamara 2006: 72–73)

Worlds can only be individuated holistically, top-down – a price to pay for the upholders of P11.