A Defense of the A-Theory of Time - *Fabrice Correia and Sven Rosenkranz*

1. The A- and the B-Theory of Time; Standard A-Theory and Standard B-Theory

The A-series and the B-series of time. B-theory: the B-series is real, the A-series is not. A-theory: the A-series is real.

The reducibility interpretation: for B-theorists, statements about the A-series can be reduced to statements about the B-series but not *vice versa*; for A-theorists, statements about the A-series cannot be reduced to statements about the B-series.

Basic A-statements:

Time *t* is present
Time *t* is *n* days ago
Time *t* is *n* days hence

Presently, *Rab*... ← |
n days ago, *Rab*... ← | elementary A-statements
n days hence, *Rab*... ← |

Complex A-statements: built from the basic ones by means of the truth-functional connectives, the quantifiers and tense-logical operators such as 'presently', '*n* days ago' and '*n* days hence'.

Basic B-statements:

Time *t* is simultaneous with time *t'*
Time *t* is *n* days earlier than time *t'*
Time *t* is *n* days later than time *t'*

At time *t*, *Rab*... ← elementary B-statements

Complex B-statements: built from these basic ones by means of the truth-functional connectives and the quantifiers.

Both A-theory and B-theory are false given the reducibility interpretation, because:

(1) *t* is simultaneous with *t*' iff sometimes, both *t* and *t'* are present.
t is *n* days earlier than *t*' iff sometimes, *t* is *n* days ago and *t'* is present.
t is *n* days later than *t*' iff sometimes, *t* is present and *t'* is *n* days hence.

At *t*, *Rab*... iff sometimes, *Rab*... and *t* is present.

(2) '*t* is present' is true at *t*' iff *t* is simultaneous with *t*'.
'*t* is *n* days past' is true at *t*' iff *t* is *n* days earlier than *t*'.
'*t* is *n* days hence' is true at *t*' iff *t* is *n* days later than *t*'.

'Presently, *Rab*...' is true at *t* iff at *t*, *Rab*...
'*n* days ago, *Rab*...' is true at *t* iff at some time *n* days earlier than *t*, *Rab*...

'*n* days hence, *Rab*...' is true at *t* iff at some time *n* days later than *t*, *Rab*...

'Presently ϕ' is true at *t* iff ϕ is true at *t*.
'*n* days ago ϕ' is true at *t* iff ϕ is true at some time *n* days earlier than *t*.
'*n* days hence ϕ' is true at *t* iff ϕ is true at some time *n* days later than *t*.

Another characterization of the divide: (i) the A-theorist countenances tensed facts, while the B-theorist reject such facts and holds that reality is constituted by tenseless facts only; (ii) the B-theorist holds that the truth-value of both A- and B-statements is determined by tenseless facts; while for the A-theorist, the truth-value of (some of) these statements irreducibly depends upon tensed facts; (iii) in particular, for the A-theorist:

(A) True utterances of elementary A-statements state tensed facts, and only tensed facts, and these are what makes the true utterances of elementary A-statements true,

while for the B-theorist:

(B) True utterances of elementary B-statements, as well as of elementary A-statements, state tenseless facts, and only tenseless facts, and these are what makes the true utterances of elementary A- and B-statements true.

Elementary A-facts and elementary B-facts: those involved in (A) and (B), respectively.

Standard A-theorists accept:

(Aa) An elementary A-fact is momentary, it obtains only at one time.
(Ab) An elementary A-fact has a distinctive intrinsic "pastness", "presentness" or "futurity" aspect whenever it obtains.
(Ac) There is no time *t* and no elementary A-fact *f* such that *f* has two distinct tensed aspects at *t*.
(Ad) There are no times *t* and *t*' and no elementary A-fact *f* such that *f* has a tensed aspect at *t* and a distinct tensed aspect at *t*'.

And for *u* an utterance of an elementary A-statement made at a time *t*:

(Ae) If *u* states a fact at some time, then that fact is an elementary A-fact.
(Af) *u* is true iff *u* states some fact at *t*.
(Ag) If *u* states a fact at some time, then *u* states a fact at *t*.
(Ah) If *u* states a fact *f* at *t*, then for every time *t*' and fact *f*', if *u* states *f*' at *t*', then *f*' = *f*.
(Ai) If *u* is of 'presently ϕ' and states *f* at *t*, then *f* has the aspect of presentness at *t*.
If *u* is of '*n* days ago ϕ' and states *f* at *t*, then *f* has the aspect of *n*-days-pastness at *t*.
If *u* is of '*n* days hence ϕ' and states *f* at *t*, then *f* has the aspect of *n*-days-futurity at *t*.

B-theorists accept:

(Ba) An elementary B-fact is eternal, it obtains at all times.
(Bb) An elementary B-fact never has a distinctive intrinsic "pastness", "presentness" or "futurity" aspect.
(Bc) If an utterance of an elementary B-statement states a fact at some time, then it states that fact at all times.

For *u* an utterance of an elementary B-statement:

(Bd) if *u* states a fact, then that fact is an elementary B-fact.
(Be) *u* is true iff *u* states some fact.
(Bf) if *u* states a fact *f*, then for every time *t*' and fact *f*', if *u* states *f*', then *f*' = *f*.
(Bg) if *u*' is an utterance of the same statement, then for every fact *f*, *u* states *f* iff *u*' states *f*.

And also:

(Bh) If an utterance of elementary B-statement 'At *t*, ϕ' states fact *f*, then any utterance of
- 'presently ϕ' made at *t* states *f*.
- '*n* days ago ϕ' made *n* days after *t* states *f*.
- '*n* days hence ϕ' made *n* days before *t* states *f*.

If an utterance of elementary A-statement 'presently ϕ' made at *t* states *f*, then any utterance of
- 'At *t*, ϕ' states *f*.
- '*n* days ago ϕ' made *n* days after *t* states *f*.
- '*n* days hence ϕ' made *n* days before *t* states *f*.

If an utterance of elementary A-statement '*n* days ago ϕ' made at *t* states *f*, then any utterance of
- 'At *t*-*n*, ϕ' states *f*.
- 'presently ϕ' made *n* days before *t* states *f*.
- '*m* days hence ϕ' made *m*+*n* days before *t* states *f*.

If an utterance of elementary A-statement '*n* days hence ϕ' made at *t* states *f*, then any utterance of
- 'At *t*+*n*, ϕ' states *f*.
- 'presently ϕ' made *n* days after *t* states *f*.
- '*m* days ago ϕ' made *m*+*n* days after *t* states *f*.

Bad consequence of standard A-theory: one and the same fact can be stated by two utterances of elementary A-statements only if they are made at the same time (and are of A-statements containing the same tense-logical operator). **B-theory does not have that consequence**.

2. A Non-Standard A-Theory

Aim: an A-theory which (i) does not have the consequence of standard A-theory, (ii) gives an account of truth for both A- and B-statements and their utterances in terms of tensed facts only.

a. A Metaphysics of Elementary A-facts

Basic principles (version with quantification over times; can be formulated without it, in a tensed language.)

(Ca) An elementary A-fact is eternal, it obtains at all times.

(Cb) An elementary A-fact has a distinctive intrinsic "pastness", "presentness" or "futurity" aspect at each time at which it obtains.
(Cc) There is no time at which an elementary A-fact has two distinct tensed aspects.
(Cd) Each elementary A-fact has presentness at some time.
(Ce) For every time t and number n, if at t, f has presentness, then at $t + n$ days, f has n-days-pastness and at $t - n$ days, f has n-days-futurity.

Fact as composed by properties or relations, objects, a tensed aspect.

b. Truth-Conditions I: A First Try

For A-statements at-a-time, use:

- 'Presently *Rab*...' is true at $t \equiv \exists f$ ($f \div <R\text{-ing},a,b,...>$ and at t, f has presentness).
- 'n days ago *Rab*...' is true at $t \equiv \exists f$ ($f \div <R\text{-ing},a,b,...>$ and at t, f has n-days-pastness).
- 'n days hence *Rab*...' is true at $t \equiv \exists f$ ($f \div <R\text{-ing},a,b,...>$ and at t, f has n-days-futurity).

- 't is present' is true at $t' \equiv \exists f$ (at t, f has presentness and at t', f has presentness).
- 't is n days ago' is true at $t' \equiv \exists f$ (at t, f has presentness and at t', f has n-days-pastness).
- 't is n days hence' is true at $t' \equiv \exists f$ (at t, f has presentness and at t', f has n-days-futurity).

- 'Presently ϕ' is true at t $\equiv \phi$ is true at t.
- 'n days ago ϕ' is true at t $\equiv \phi$ is true n days before t.
- 'n days hence ϕ' is true at t $\equiv \phi$ is true n days after t.

For B-statements, use the equivalences in (1). For utterances: an utterance of an A- or B-statement is true iff the statement is true at the time of utterance.

We are left with reference to / quantification over times, in expressions of type '--- is true at t', 'at t, f has presentness' and 't is n days before t", we still have reference to times. **PROBLEM**: the latter statements are among the B-statements.

c. The Reduction of Times

Facts f and g are contemporary (f C g) iff either both f and g have presentness, or there is an n such that both f and g have n-days-pastness, or there is an n such that both f and g have n-days-futurity.

C has a rigid extension, and is an equivalence relation. Identify times with classes of equivalence for C.

Fact f is n days before fact g (f n-< g) iff either f has presentness and g has n-days-futurity, or f has n-days-pastness and g has presentness, or there are two numbers u and v such that
- $u+v=n$ and f has u-days-pastness and g has v-days futurity
- $v-u=n$ and f has u-days-futurity and g has v-days futurity
- $u-v=n$ and f has u-days-pastness and g has v-days pastness.

Relation n-< is rigid, and preserved under C. Identify the temporal order with it.

Replace 'at t, f has presentness' by '$f \in t$', 'at t, f has n-days-pastness' by '$\exists t'$ (t' n-< t and $f \in t'$)', and 'at t, f has n-days-futurity' by '$\exists t'$ (t n-< t' and $f \in t'$)'.

c. Truth-Conditions II: A Second Try

Truth-at-a-time for A-statements:

- 'Presently *Rab*...' is true at $t \equiv \exists f\,(f \div \langle R\text{-ing},a,b,...\rangle$ and $f \in t)$.
- '*n* days ago *Rab*...' is true at $t \equiv \exists f\,(f \div \langle R\text{-ing},a,b,...\rangle$ and $\exists t'\,(t'\ n\text{-}<\ t$ and $f \in t')$.
- '*n* days hence *Rab*...' is true at $t \equiv \exists f\,(f \div \langle R\text{-ing},a,b,...\rangle$ and $\exists t'\,(t\ n\text{-}<\ t'$ and $f \in t')$.

- '*t* is present' is true at $t' \equiv \exists f\,(f \in t$ and $f \in t')$.
- '*t* is *n* days ago' is true at $t' \equiv \exists f\,(f \in t$ and $\exists t'\,(t'\ n\text{-}<\ t$ and $f \in t'))$.
- '*t* is *n* days hence' is true at $t' \equiv \exists f\,(f \in t$ and $\exists t'\,(t\ n\text{-}<\ t'$ and $f \in t'))$.

- 'Presently ϕ' is true at t ≡ ϕ is true at *t*.
- '*n* days ago ϕ' is true at t ≡ ϕ is true *n* days before *t*.
- '*n* days hence ϕ' is true at t ≡ ϕ is true *n* days after *t*.

For truth (*simpliciter*) for B-statements, use the equivalences in (1). For utterances: an utterance of an A- or B-statement is true (*simpliciter*) iff the statement is true at the time of utterance.

Tensed truth for A-statements (prefix with 'Always'):

- 'Presently *Rab*...' is true ≡ $\exists f\,(f \div \langle R\text{-ing},a,b,...\rangle$ and *f* has presentness).
- '*n* days ago *Rab*...' is true ≡ $\exists f\,(f \div \langle R\text{-ing},a,b,...\rangle$ and *f* has *n*-days-pastness).
- '*n* days hence *Rab*...' is true ≡ $\exists f\,(f \div \langle R\text{-ing},a,b,...\rangle$ and *f* has *n*-days-futurity).

- '*t* is present' is true ≡ $\exists f\,(f \in t$ and *f* has presentness).
- '*t* is *n* days ago' is true ≡ $\exists f\,(f \in t$ and *f* has *n*-days-pastness).
- '*t* is *n* days hence' is true ≡ $\exists f\,(f \in t$ and f has *n*-days-futurity)).

- 'Presently ϕ' is true ≡ ϕ is true.
- '*n* days ago ϕ' is true ≡ *n* days ago, ϕ is true *n* days before *t*.
- '*n* days hence ϕ' is true ≡ *n* days hence, ϕ is true.

For truth for B-statements, use the equivalences in (1). For utterances: An utterance *u* of an A- or B-statement *S* is true iff sometimes, *u* occurs and *S* is true.