

Logical Unity: Wittgenstein's Challenge, Frege's Answer

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Die scharfe Scheidung von Ergänzungbedürftigem und Gesättigtem ist sehr wichtig.

Gottlob Frege

1. Introduction

According to Frege, the realm to which logic applies, namely everything, is divided into the complete or saturated and the incomplete or unsaturated. Sentences have complete and incomplete constituents. The complete constituents, *proper names*, have complete senses and referents, *objects*; the incomplete constituents, *function words*, have incomplete senses and referents, *functions*.¹ *Concepts* are those functions which map any object to one of the truth-values, but never to both or neither. In 'Compound Thoughts' Frege goes on to formulate a general hypothesis:

(Frege's Hypothesis) [I]t is natural to suppose that, for logic in general, combination into a whole always comes about by the saturation of something unsaturated. (CP, 390).

Wittgenstein and Ramsey don't find this natural at all. Both deny that the constituents of sentences, circumstances and thoughts can be divided into saturated and unsaturated ones. Every constituent of such a complex is unsaturated.

Philosophers have tried to meet the Wittgenstein/Ramsey challenge in different ways and in doing so raised new challenges.² In this paper I will not evaluate the responses presented in the literature, but provide a new response that clarifies what is meant by "unsaturatedness".

¹ See PW, 243

² For detailed alternative attempt to meet the challenge see Strawson 1959, part II and Geach 1962. Geach 1962 § 20 argues that proper names can be used 'in isolation'. His proposal is symmetrical to the one presented here. Hale 2006 presents a more recent attempt. MacBride 2005a and 2005b convincingly criticise the responses to the Wittgenstein/Ramsey challenge in the literature.

2. Logical Composition and Ontic Unsaturatedness

In order to assess the ‘naturalness’ of Frege’s hypothesis we need to know more about what it is to be unsaturated.

From 1879 to 1891, Frege identifies judgeable contents, what we take to be so when we judge, with complexes of particulars and concepts. These complexes are either true or false, but Frege has not yet introduced the truth-values as objects. Concepts are characterised as unsaturated. Frege writes in a letter from 1882:

[A] A concept is unsaturated in that it requires something to fall under it; hence it cannot exist on its own. That an individual falls under it is a judgeable content, and here the concept appears as a predicate and is always predicative.

[B] In this case, where the subject is an individual, the relation of subject to predicate is not a third thing added to the two, but it belongs to the content of the predicate which is what makes the predicate [unsaturated]. Now I do not believe that concept formation can precede judgement because this would presuppose the independent existence of concepts, but I think of a concept as having arisen by decomposition from a judgeable content. (PMC, 101)

Let us start with part *A* of the quote, I will devote the next section to *B*.

In *A* Frege characterises the unsaturatedness of concepts as a distinctive form of ontic dependence. Every traditional logician will say that concepts subsume things. Frege starts from this uncontroversial point of folk logic and expands it to a controversial claim:

(*Ontic Unsaturatedness*) The unsaturatedness of a concept consists in the fact that it cannot exist without subsuming something.

Concepts are entities, argues Frege, that cannot exist in isolation from other things they subsume, while objects can:

It is clear that we cannot present a concept as self-subsistent, like an object; rather it can occur only in combination. One may say that it can be distinguished within but not separated from this combination. (KS, 270; FG, 34)

A begriffsschrift sentence signifies a state of affairs, a combination of the contents of the proper names and function words composing the sentence. An ontologically sound

representation of a state of affairs must portray concepts ‘saturated’ by an object. For example, “(John) smokes” correctly represents the circumstance signified.

The obvious objection to (OU) is that some concepts don’t subsume anything and it is unclear in which sense a concept like “ $\sim (x = x)$ ” requires that something falls under it. Nothing falls under it; its extension is the empty set.³

After the introduction of truth-values Frege is committed to saying that every concept maps every object either to the True or the False. This points us towards an answer to the objection just made. Concepts are, so to say, mediators between objects and truth-values. Furthermore, there is nothing more to a concept than this. Hence, a concept cannot exist without other things.

Let us assume that an atomic circumstance consists of a function and an argument. Why should one say that a function cannot, while the argument can, exist in isolation? If there is an asymmetry between objects and concepts in this point, objects must be able to exist in isolation. But an object a cannot exist without being the constituent of a range of facts, although a need not be a constituent of any particular fact in this range. If a can exist in isolation, that is, without being a constituent of any circumstance or fact, having no properties, a would be a bare particular. This consequence is an unwelcome commitment; bare particulars are rather bizarre ‘things’. Moreover, there are certain properties which a necessarily has. If a is a human being, a is necessarily human. a cannot exist without occurring in the fact that a is human. Hence, neither objects nor functions can exist in isolation.

If both object and concept are ontically unsaturated, neither can be the sole source of logical unity. Wittgenstein illustrates the resulting view of logical unity with his chain metaphor: in the atomic fact objects hang one in another, like the links in a chain. (See TLP 2.03) Similarly for the unity of a sentence and the thought

In the *Tractatus* Wittgenstein uses the notion of essence to make his point: as it is essential for concepts to occur together with objects in states of affairs, it is essential for objects to occur together with concepts in states of affairs

I will now set ontic unsaturatedness aside and return to it at the end of the paper. More important is what Ramsey calls ‘special unsaturatedness’.

³ For a different characterisation of concepts in terms of subsumption see FA, §51.

3. Special Unsaturatedness

In part *B* of the quotation from Frege's letter to Marty, Frege uses the linguistic notion of a predicate to say what makes concepts unsaturated. Concepts appear "as predicates" in judgeable contents. With atomic contents in mind he argues

(Special *Unsaturatedness*) The unsaturatedness of a predicate consists in the fact that the content of a predicate contains the relation between subject and object.

Predicates are unsaturated because they contain the subject-predicate combination, i.e. an atomic sentence. Now a predicate will not contain a particular subject-predicate sentence, but one can say, as a first stab, that it contains the form of an atomic sentence.

Russell will make a similar point some 30 years later. When you understand a proper name

you do not, that is to say, have any suggestion of the form of a proposition, whereas in understanding a predicate you do. To understand 'red', for instance, is to understand what is meant by saying that a thing is red. ... Exactly the same applies to relations, and in fact to all those things which are not particulars. (PLA, 205)⁴

Special unsaturatedness is one of the targets of the Wittgenstein/Ramsey challenge. In the *Tractatus* Wittgenstein extends his challenge to expressions:

3.3 Only propositions have sense; only in the nexus of a proposition does a name have meaning.

3.31 I call any part of a proposition that characterizes its sense an expression (or a symbol). (A proposition is itself an expression.) Everything essential to their sense that propositions can have in common with one another is an expression. An expression is the mark of a form and a content.

3.311 An expression presupposes the forms of all the propositions in which it can occur. It is the common characteristic mark of a class of propositions.

3.312 It is therefore presented by means of the general form of the propositions that it characterizes. In fact, in this form the expression will be constant and everything else variable.

⁴ For a more recent proposal along similar lines, see Burge 2007, 599.

3.3 is an echo of Frege's context-principle. It is distinctive of Frege's approach to logic that it is top-down: Frege starts with thoughts and sentences and decomposes them. It is for this reason that Frege describes his conception of logic retrospectively in the following way:

I do not begin with concepts and put them together to form a thought and judgement; I come by the parts of a thought by analysing the thought. This marks off my *begriffsschrift* from the similar inventions of Leibniz and his successors, despite what the name suggests; perhaps it was not a happy choice on my part. (PW, 253)

Frege's top-down approach is the basis for his solution to logical problems but it is also the basis for the challenge mounted by Wittgenstein.⁵ Let us spell this out.

Take again the sentence 'John smokes'. What entitles us to take 'smokes' to contain the form of an atomic sentence and not 'John'? 'John' can be said to contain the form of the sentence as does 'smokes'. 'John' is a shared constituent of many sentences:

John drinks.
John is happy.
John loves Mary.

So why not take 'John' to be adequately represented by 'John { }', where '{ }' is a variable that can only be completed by predicates to form sentences? Predicates, in turn, have a gap that can only be filled by proper names '() smokes'. In this way, the variable part of an expression is distinctive of its logical type. If we take both types of constituents to be unsaturated, an atomic sentence looks like this:

(John) {smokes}

Proper names and predicates hang in each others 'gaps' like the links in a chain. There is no designated element in a sentence that is complete and another that is incomplete. Both proper name and predicate have the potential to form a sentence and they share, so to say,

⁵ See Kremer ms, 24-5.

the logical form of the sentence. All words are incomplete.⁶ Wittgenstein seems just to draw the natural conclusion from Frege's distinctive top-down approach to logic.⁷

Ramsey repeats Wittgenstein's challenge with Russell in mind:

The great difficulty with this theory lies in understanding how one sort of object can be especially incomplete. There is a sense in which any object is incomplete; namely that it can only occur in a fact by connection with an object or objects of a suitable type; just as any name is incomplete, because to form a proposition we have to join to it certain other names of suitable type. (Ramsey 1927, 63, see also 59)

In short, Wittgenstein and Ramsey argue:

Proper names (in a broad sense) and concept words are both constituents of assertoric sentences without being themselves sentences.

Therefore, proper names and concept words are both incomplete.

Wittgenstein's and Ramsey's *modus ponendo ponens* is my *modus tollendo tollens*: If there is a sense of 'incomplete' in which all constituents of a sentence (states of affairs) are incomplete, this cannot be the sense Frege takes to be logically fundamental. Frege must take special unsaturatedness to be a property that is different from *being a subsentential constituent of an assertoric sentence*. Frege and Russell have already pointed at the special variety of unsaturatedness by saying that an unsaturated expression contains the relation between subject and predicate. If this *modus tollendo tollens* shall be a convincing reply to Wittgenstein's challenge, we must give this idea more substance than Frege himself has done. The next sections will try to do so. In the next section I will

⁶ See Ramsey 1927, 63. See also 59 and Linsky 1992, 267.

⁷ The tension between his top-down approach and his characterisation of object and proper names as saturated did not escape the author of *Foundations of Arithmetic*: "The self-subsistence which I claim for number is not taken to mean that a number word signifies something when removed from the context of a sentence, but only precludes the use of such words as predicates of attributes, which appreciably alters its meaning." (FA, §60) It is difficult to see how this can be a good answer to the challenge Frege implicitly acknowledges. For Frege presupposes that only predicates are unsaturated and stand for dependent 'things', while this is just what has to be justified.

argue that one cannot defend Frege if one explains special unsaturatedness merely negatively. In the following sections I will present a new account of what concept words, gaps and variables are.

4. The Deletion View of Linguistic Unsaturatedness

Let us start with an outline of the distinctive feature of an unsaturated sign: it contains a gap. Frege takes talk of gaps literally:

We can start from the sentence “ $3 - 2 > 0$ ”. We decompose it into the proper name “ $3 - 2$ ” and the remainder “ > 0 ”. One can say that the unsaturated part refers to the concept of a positive number.” (NS, 212. My translation.)

If an expression is a remainder of sentence after deletion of a proper name, it is unsaturated. I will call the thesis that an unsaturated part of a sentence is a remainder, *the deletion view of (linguistic) unsaturatedness*. Dummett argues that it is the conceptual basis of Frege’s theory about statements of multiple generality (“In every village there is one girl loved by every boy’):

Frege’s insight consisted in considering the sentence as being constructed in stages, corresponding to the different signs of generality occurring in it. A sentence may be formed by combining a sign of generality with a one-place predicate. *The one-place predicate is itself to be thought of as having been formed from a sentence by removing one or more occurrences of some singular term (proper name)*. Thus we begin with a sentence such as ‘Peter envies John’. From this we form the one-place predicate ‘Peter envies ξ ’ by removing the proper name ‘John’ – the Greek letter ‘ ξ ’ here serving merely to indicate where the gap occurs that is left by the removal of the proper name. This predicate can then be combined by with the sign of generality ‘somebody’ to yield the sentence ‘Peter envies somebody’. (Dummett 1980, 10-11. My emphasis.)

Dummett is a bit too quick: Frege’s unsaturated concept words differ from the predicates of predicate logic. In the predicate “ $x > 0$ ”, “ x ” can be bound by a quantifier. The “ x ” (i) marks and makes a gap in an expression and (ii) it is bindable by a quantifier (“ $(\exists x) (x = 3 + 5)$ ” is a well-formed sentence/formula). But the deletion view has it that “ $\xi > 0$ ” or “ $() > 0$ ” is short for “ $\dots > 0$ ”. The *gap* in “ $() > 0$ ” cannot be bound by a quantifier. The most fitting way to write a concept word is “ $() = 3 + 5$ ”, because this expression has really a

gap between the brackets which can be taken by argument expressions.⁸ Less fitting is “ $\xi=3+5$ ”, in which “ ξ ” marks a gap.

If the gap in “ $() > 0$ ” cannot be bound by a quantifier, how does one make general statements in Fregean logic? By closing the gap in a function or concept word with a letter: “ $a = 3 + 5$ ”. Letters can close gaps because they have semantic properties: they don’t refer to an object, but they indicate indeterminately an object (“unbestimmt andeuten”).⁹ What does it mean to say that a sign indeterminately indicates an object?

The adverb “indeterminately” provides the clue. Take “ a ” to range over numbers. Each number is as good a candidate as any for being the referent of “ a ”. The meaning of “ a ” does not therefore pick out one of them as the referent of the term. But it also does not pick out the moon as the referent. It stands indeterminately for numbers. As in other cases of indeterminacy, we can sharpen the sense of the sign. This sharpening is effected by giving the letter the sense of a particular proper name of number. If a formula is true under all sharpenings of the letters it contains, it is universally true and this ensures that validity of universal instantiation. The letter itself does not stand for any number determinately, it indicates numbers indeterminately.¹⁰

Is ‘indeterminate indication’ a form of ambiguity?¹¹ Sometimes it seems so. A letter can indeterminately indicate many things if either its sense is unspecific or it has many senses. In both cases there are many candidates for being its referent. Doesn’t Frege’s requirement that a scientific language cannot contain ambiguous expressions tell against the ambiguity reading? No, for these strictures concern only expressions with determinate sense and determinate reference, but letters have no reference, they only indeterminately indicate.

I have belaboured the distinction between a placeholder for argument expressions like “ ξ ” and generality bestowing letters like “ a ” at length because having the distinction in place necessary if one is to understand Frege’s doctrine of special unsaturatedness properly. According to Frege, “ $() > 0$ ” or “ $\xi > 0$ ” are incomplete and stand for the unsaturated concept. By contrast, “ $a > 0$ ” is not a name of a concept, but an indeterminate indication of a value of the concept that “ $\xi > 0$ ” stands for.¹²

⁸ See ,What is a Function?’, 664.

⁹ See BS, § 1, ‘What is a function?’, 660.

¹⁰ For alternatives to Frege’s semantics of variables see Fine 1987 (arbitrary objects) and Lance 1996, 490 (arbitrary names).

¹¹ The following paragraph addresses worries raised in MacBeath 2005, 61.

¹² See NS; 258. See also letter to Dingler, WB, 34.

In the following section I will be mainly concerned to find a better understanding of “ $\xi > 0$ ” than the one Frege himself provides. However, we need to bring in Fregean letters to address Linsky’s often repeated objection against Frege’s hypothesis:

In the case of second-level concepts, such as quantifiers, the only way to deploy these metaphors [saturation and unsaturatedness] is to characterise the application of function to argument as the completion of an incomplete thing with another incomplete thing, or the saturation of an unsaturated thing with another unsaturated thing. It is as though putting one unsaturated sponge together with another would produce a saturated pair of sponges. The metaphors have gone entirely limp: they cast no light whatsoever. (Linsky 1992, 265)

Linsky’s objection assumes that a statement of generality is completely constituted by a first-order concept saturating a second-order concept. (“There is at least one square root of 4”). However, a look at Frege’s rendering of statements of generality in *Begriffsschrift* shows that such statements also contain letters that close the gaps in the first level concepts. (Roughly “Not (a) is not a square root of 4”).)

5. Problems for the Deletion View

I will now use the problems of the deletion view as pointers to a better conception of the special unsaturatedness of concept words.

If a concept word is really just a sentence remainder, it cannot refer to a concept nor can it have any other referent. The reason is that the sentence remainder carries no information that it has been generated by deleting a proper name and that it needs filling by a proper name to generate a new proper sentence. For the remainder “ > 0 ” can be completed in numerous ways to a true sentence. Consider the following examples:

“John believes that 7” saturates “ $\dots > 0$ ” to “John believes that $7 > 0$ ”

“Either $1 < 0$ or 1 ” saturates “ $\dots > 0$ ” to “Either $1 < 0$ or $1 > 0$ ”

Since “ > 0 ” can be saturated in many ways, Frege’s assumption that it stands for the concept of a positive number is unjustified and unjustifiable. If the deletion view were correct, “ > 0 ” would not stand for a function from objects into the truth-values.

Conclusion: one cannot assign to a sentence remainder any semantic value if the gap does not restrict what can fill it.

No wonder that for the deletion view of unsaturatedness Wittgenstein's challenge is unanswerable. If a predicate is nothing more than the remainder of a sentence after removal of at least one proper name, it will be impossible to justify the view that concept words are especially incomplete. For proper names are the remainder of sentences after removal of predicates. Both are remainders, both are incomplete, both contain the form of the sentence.¹³ If one starts with wholes and deletes or subtracts from them constituents, as Frege himself proposes, the asymmetry between complete and incomplete things cannot be justified. In the next section I will try to do better.

For these reasons one should replace the deletion view of incompleteness with a better one. Frege helps himself to such a view when he writes:

This “ξ” is no part of the function name, but it only serves to make recognisable how the function sign connects with completing proper name. *This “ξ” provides us with a manual [Gebrauchsanweisung] for the function name.* (WB, 259. My translation and emphasis.)

The placeholder “ξ” can only be a manual with instructions for how to complete the function expression if it is more than a gap, a mere absence. But what sign can

make a gap in a sentence,
and
constrain which expressions fill the gap to (true) sentences
without
being bindable by quantifiers?

6. The Interrogative View of Incompleteness and Cohen's Suggestion

The deletion view assumes that one decomposes sentences into proper names and letters and concept or function words. We can make progress with our question if we ask why one should assume that sentences are decomposed into these parts. Frege's answer is that judgement distinguished parts in conceptual content. In his mature work judgment is supposed to distinguish parts in a thought and indirectly in a truth-value.¹⁴ But this pushed the problem only one step back: Why does judgment decompose content at all and if so, why should it decompose it into saturated and unsaturated parts?

¹³ Frege himself proposes such a view of incompleteness in § 70 of FA.

¹⁴ CP 164, OP, 35.

In answering these questions I will focus on assertion, the linguistic manifestation of judgement. Philosophers usually start to understand assertion independently of other speech acts. Questions and imperatives are fitted in later. Now this might get things the wrong way around:

Most assertions are responses to questions. If I simply volunteer a piece of information for no apparent reason, it seems crazy. A context is needed which makes the assertion relevant. Perhaps I have been asked an explicit question and my assertion is intended as an answer to it. Perhaps the question is implicit from context: we are all aware that a question is in the air, so you interpret my utterance as a response to it. But unless there is a question, a real question, to which the assertion is an answer, it makes no sense. Hence, *rather than seeing the question containing (implicitly) a proposition, we might (with more justice) see the assertion as containing a question.* (Hookway 1996, 9. My emphasis)

Hookway's last remark is suggestive, but also puzzling. How can a question implicitly contain a proposition? If a question contained a proposition, one should think that one could not ask a question without already having the answer to hand. Clarifying this point will help to understand special incompleteness.

Hookway does not endorse the claim that *every* assertion is a response to a question. Some assertions may be spontaneous. (I utter with assertoric force "This car is really fast" after I am overwhelmed by the acceleration of my new Maserati). How can the assumption that assertions are answers to questions then help?

There are different responses to this question. One is that an assertion that is not an answer to a question will not be decomposable into parts as Frege supposes. (Below I will explain how questions can help us to understand decompositions of assertions.) One can of course decompose the asserted thought and sentence in standard grammatical categories, but Frege frequently proposes decompositions which don't follow grammar.

Another reply is that if an assertion aims to impart knowledge the utterer must exclude relevant possibilities and that the best account of which possibilities have to be excluded appeals to questions. The utterer must exclude those possibilities which are under question in the discourse. Hence, the fundamental norm of correctness for assertion appeals to questions.¹⁵

¹⁵ See Schaffer MS.

In this paper I will rely on the first point and continue working with the idea that all interesting assertions are answers to questions.

Frege himself is sympathetic to the view that assertions are answers to questions. In general, questions play an important role in Frege's philosophy. For example, a concept is something for which *the question arises whether something false under it*; a particular is something for which this question makes no sense, but *the question of its identity with something arises*; a thought is something for which *the question of truth arises*.

In 'Thought' Frege connects word-questions, assertion and the saturated/unsaturatedness distinction:

In a word-question we utter an *incomplete* sentence, which is meant to receive a true sense just by means of the *completion* for which we are asking. (CP, 355 (62). My emphasis)

In the following section I will try to exploit the idea formulated by Frege to shed light on the special unsaturatedness of concept words. Let us first make some helpful distinctions. One must distinguish between the speech-act of *asking a question*; its product, the *question asked*, and finally the *interrogative sentence* by means of which the question is asked. Frege calls the speech act made by means of an utterance of an interrogative sentence that contains interrogative pronouns like 'who', 'what', 'where' and 'when' a 'word-question'. These contrast with propositional questions that can be answered with 'Yes' or 'No'. I will call interrogative sentences with interrogative pronouns "wh"-sentences.

Frege moves on from the view that word-questions are incomplete to the view that word-questions are requests for the completion of a sentence remainder. The question "Who contradicts Hegel?" is the request to complete "... contradicts Hegel" to a true sentence! If questions are requests for completing the gaps in a sentence fragment to generate a true sentence, we don't now how to answer the question asked. The request for completion to a true sentence puts no constraints on what may effect the completion.

I propose therefore to take interrogative sentences with interrogative pronouns to be more than sentence fragments. Here is a model. If you visit the office of some companies you have first to fill out a little form. The form can look like the following one:

Name	
Company	
Reason for Visit	

Your giving me the form gives me a reason to fill it out; it is asking me a question. When I fill out the form, and there are no defeating factors, I have answered your questions.

Is asking a word-question then requesting the completion of a propositional function as Cohen's proposal suggests?

A question, it is submitted, is simply a propositional function (or propositional form). "What is the sum of 3 and 5?" seems identical in logical content with " $x = 3 + 5$ " (Cohen 1929, 353)

And:

Who, which, what, when, where, why, etc. are the variables of every-day speech. (ibid., 354)

Prima facie, interrogative pronouns are not variables. An interrogative pronoun cannot be bound by a quantifier. For example, one cannot substitute *salva congruitate* for the variable "x" in "For every x, the sum of 3 and x is greater than the sum of 2 and x" an interrogative pronoun. We build quantifiers from interrogative pronouns ("Whoever killed Smith is insane"), but interrogative pronouns are not bindable by quantifiers.

Hiz has responded to this problem by maintaining that interrogative pronouns are variables and introducing a new piece of language: 'a questioner'. Every word-question consists of a questioner and a propositional function: "For which number x: x is the sum of 3 and 5".¹⁶ Linguists have followed Hiz and render the syntactic form of word questions as "[wh_x [...x...]_s]_q".

There is no need to introduce questioners. We can take word-questions for what they seem to be: incomplete sentences in which interrogative pronouns play the role of Fregean placeholders and an extra bit more. Which extra-bit more? Our model suggests that the interrogative pronouns mark gaps in the interrogative sentence and understanding an interrogative pronoun consists in part in knowing what can fill the gap.

¹⁶ Hiz 1962, 255. Hiz credits Ajdukiewicz with the original idea.

First, interrogative pronouns hold open places into which other expressions can enter. They mark, as Frege says, the place where other expressions can enter.

Second, interrogative pronouns not only mark argument places, they contain

(i) syntactic information about which kind of expressions can enter the place. For example, the “wh”-sentence “Who is fat?” cannot be completed by “Peter is tall and he”. The interrogative pronoun must be replaced by singular terms that name objects of a contextually salient range. Moreover, interrogative pronouns contain case information (“For whom does the bell toll?”). They are indeed manuals for the completion of concept words.

(ii) sortal information about the argument: “where” requires completion by designators of locations, “when” by designators of times, etc.

Is every interrogative sentence a function word with a certain number of empty places? Fiengo and Cohen answer: YES.¹⁷ The interrogative sentence “Is John fat?” contains an interrogative word that is not visible in the surface. Frege answers NO. I will not enter the debate in this talk. For my purposes it is sufficient to argue that “wh”-sentences are the prototypes of Fregean concept-words.

Is every function word an interrogative sentence? *Prima facie*, NO. Only Fregean concept words (Russellian propositional functions) are interrogative sentences. What about functional expressions like “ $2 \cdot ()^3 + ()$ ” whose correct completion is a singular term? In line with Frege’s maxim to start with sentences and thoughts, they are parts of interrogative sentences. For example, we start with “What is the sum of 2 plus 5?” then generalise further “What is the sum of what plus which?” and finally decompose the question into parts one of which is “what plus which” or in Frege’s “ $\xi + \zeta$ ”

8. Meeting the Challenge

We are now in a position to meet the challenge posed by Wittgenstein and Ramsey. For we have good reasons to endorse

(SU1) Concept words are especially incomplete in that they contain *the form of a specific kind* of assertoric sentence.

If a concept word is a ‘wh’-sentence it is a framework for the generation of assertoric sentences that can be used to answer the question posed by an utterance of the ‘wh’-sentence. Understanding the ‘wh’-sentence puts one in a position to sort assertoric

¹⁷ See Fiengo 2007, 7.

sentences one understands into those that are and those that are not candidates for a correct answer. Otherwise why should one ask a question if one would not know what would constitute an answer to it? If you understand the interrogative sentence “Who is fat?”, you know, among other things, which words in which order will constitute an answer to a question posed by means of it. The so-called *proposition set semantics* for questions illustrates this point: the meaning of a question is the set of all answers to it. This is too simple, but suggests the right view.¹⁸

We have also a good reason to stress the following aspect of Frege’s thesis:

(SU1*) Concept words are especially incomplete in that they contain the form of a specific kind of *assertoric* sentence.

Why? If I use a “wh”-sentence to ask a question, I conventionally indicate that my knowledge has a gap.¹⁹ The interrogative sentence serves to pinpoint exactly what I fail to know and how the lack of knowledge can be remedied with the help of a reliable source of information. The simple method to close the gap in my knowledge is to make an assertion of an assertoric sentence in which the interrogative pronouns are replaced by suitable expressions. If the assertion is made from knowledge, and there are no counter-indications, I come to know what I did not know before. Only an assertion, an utterance that makes a truth claim, will do. Hence, “wh”-sentences contain the form of an assertoric sentence. Someone who understands a “wh”-sentence will know that an assertion of a sentence in which the gaps of the “wh” sentence are closed is the conventional response to an utterance of the sentence.

Let us now return to the Wittgenstein/Ramsey challenge. According to them, names as well as predicates are unsaturated because neither can occur in isolation. This may be true. However, this is not the theoretically important sense of linguistic unsaturatedness that underpins Frege’s theory of generality. For:

Proper names (in Frege’s broad sense) are NOT especially incomplete in that they do NOT contain the form of a specific kind of assertoric sentence.

¹⁸ See Hamblin 1958.

¹⁹ See Fiengo 2007. Fiengo uses Frege’s concept of incompleteness to shed light on the semantics and pragmatics of questions, I use the semantics and pragmatics of questions to shed light on Frege’s concept of incompleteness.

A singular term like “Hegel” does not contain the form of a specific kind of assertoric sentence. A use of “Hegel” does not conventionally indicate lack of a particular kind of knowledge. Hence, we have no reason to say that it suggests (Russell) the form of an *assertoric* sentence. It does not suggest the form of any kind of sentence in particular: “Hegel” can occur in question sentences, command sentences etc.

It is furthermore not clear that “Hegel” contains a sentence at all. Someone who understands a concept word knows that assertoric utterances of sentences of a particular form constitute answers to the question. By contrast, one can understand a proper name without being able to specify a particular set of sentences in which “Hegel” figures. For example, we have no independent reason to take “Hegel” as an expression whose syntactic profile is specified by saying that it returns sentences for concept words. “Hegel” also returns sentences operators for propositional attitude verbs “believes that” or plural names for plural names (“Hölderlin and Schelling and Hegel” for “Hölderling and Schelling”). We have no reason to single out assertoric sentences as the distinctive completions of proper names. We have for instance no independent reason to say that “Hölderlin and Schelling and Hegel” is not yet a completion of “Hegel”. This undermines the idea of seeing proper names as having a gap for concept words.

The previous points show that concept words aka questions contain in an uncontroversial sense assertoric sentences that complete them. Proper names don't. Wittgenstein's challenge is based on the obvious truth that proper names and concept words can be constituents of sentences. But it ignores that concept words contain the form of a sentence, proper names don't. Hence, the challenge fails to put Frege under pressure.

We can now also clarify the relation between special unsaturatedness and literal incompleteness:

Only if an expression has gaps that constrain their completers does it contain the form of an assertoric sentence, that is, is it especially unsaturated.

We can further make Frege's claim plausible that the basic unsaturated entities are senses. It is due to that fact that a sign has the sense it has that it contains the form of an assertoric sentence.²⁰ The special incompleteness of concept words consists in facts about what it is to understand these words and the occurrence of interrogative pronouns in them.

²⁰ CP, 393 (39).

Finally, special unsaturatedness is not artificially concocted only to rebut an objection. It falls out of an independently plausible understanding of judgement. Frege takes judgement to be the logically primitive activity. Investigations of judgement and assertion recommend taking judgements as responses to questions. This is sufficient to motivate the distinction between expressions and senses that specially incomplete and those that are complete.

9. How concept-words are contained in atomic sentences

According to Frege, every atomic sentence is composed out of an unsaturated function word and a proper name:

If we take “ $2 \bullet 2 = 4$ ” as resulting from the replacement of the letter “ ξ ” in “ $\xi \bullet \xi = 4$ ” by the numeral “2”, “ $2 \bullet 2 = 4$ ” appears to be composed from the name “2” and from a concept sign that is as such in need of completion, therefore one can read “ $2 \bullet 2 = 4$ ” read as “2 is a square root of 4”. (WB, 255. For singular terms see, CP, 291, 664)

Now we can also take “ $2 \bullet 2 = 4$ ” as resulting from the replacement of the letter “ $2 \bullet 2 = \zeta$ ”. We do so, if “ $2 \bullet 2 = 4$ ” is an given as an answer to the question “What is the result of multiplying 2 with itself?”. “ $2 \bullet 2 = 4$ ” reads then as ‘4 is 2 multiplied with itself’. Hence, the same sentence can have different parts. How one can read it depends on the question asked.

In which sense is a concept word contained in an assertoric sentence? There are many notions of containment. According to one, something x is contained in something y if x can be discerned or detected in y and there is an account of how x helps to constitute y . Now how can one discern “Who is fat?” in “John is fat?”

One can discern the interrogative sentence in the assertoric sentence used to answer it if one pays attention to focus. Linguists classify some question/answer pairs as congruent, others as incongruent. The following question/answer pair (Q1/A1) is OK (congruent), the pair (Q2/A1) not, although both are syntactically and semantically impeccable:

(Q1) Which number times which number is 4? (A1) $[2]_F$ times $[2]_F$ is 4.

(Q1) 2 times 2 is which number? (A1) $[2]_F$ times $[2]_F$ is 4.

(Q2/A1) is incongruent. Why? Because ‘2’ cannot be in focus given the question asked. This suggests that the expressions in focus are the ones occupying the gaps in the function word, the function word being the interrogative sentence which can be generated by substituting the expressions in focus with interrogative pronouns. Focus provides a foreground/background structure for the information provided by an utterance. The completing expression is in the foreground, the incomplete is in the background. The assertoric sentence contains the interrogative one, but it does not contain it as a quotable part. This is a contingent fact about the languages we speak.²¹ We could make assertions by combinations of questions and answers (‘Who is fat? John!’). We don’t do it and in our assertoric expressions the predicate is not a separable expression. No wonder, the unsaturated predicate has been saturated with a proper name. But it makes good sense to speak of saturation here for the unsaturated element can still be discerned, but not detached.

This understanding of constitution justifies our departure from the letter of Frege’s writings. Frege takes concept words literally to have gaps. For otherwise they could not occur in sentences. For example, the sentence “John is fat” does not contain “Who is fat?” as quotable part, but it does contain the gappy “... is fat”. However, a more relaxed and independently plausible understanding of constitution allows us to say that “Who is fat?” is contained in saturated form in “John is fat”. It is contained in the assertoric sentence because the assertoric sentence is generated by completing the gaps in the interrogative sentence correctly and the interrogative sentence can be reconstituted via question/answer congruence tests.

It is a widely held view among Fregeans that unsaturated function words are not ‘bits of language’ contained in sentences. Geach writes:

[A predicate is not] an actual expression occurring in sentences. But a common property of sentences ... The distinction between thing and property comes out in the distinction between name and predicate just because a name is an actual expression whereas a predicate is rather a common property of expressions. (Geach 1953, 224)

Dummett agrees with the crucial point:

²¹ See Cook-Wilson on sentence representation.

[A function symbol] cannot literally be removed from a sentence and ... displayed on its own: we can only indicate the common features of various sentences which we have in mind by the use, together with words or symbols belonging to the language, of the Greek letters which represent argument-places. And it is, in turn, just because the complex expression is thus not really an expression – a bit of language – in its own right, that we are to regard it as formed from a sentence rather than as built up from components. (Dummett 1980, 31)

The Dummett-Geach Argument goes as follows:

Assertoric sentences don't contain predicates because they cannot be removed from sentences.

Therefore: predicates are properties of sentences.

Therefore: a predicate cannot exist independently of some sentences.

Therefore: predicates are ontologically dependent.

If concept words are interrogative sentences with interrogative pronouns, they are bits of language in their own right. Dummett and Geach are wrong. Hence, one of the premises of their argument must be false. Which?

The first premise rests on an implausible understanding of containment. Why is removability necessary for containment? Some things are contained in others but cannot be removed from them.

The view of constitution proposed above avoids the counter-intuitive consequences that arise for the Dummett-Geach view. Since atomic sentences must be constructed from independently available words, Dummett is forced to have two kinds of predicates. For instance, the word 'loves' and a pattern 'love' which I can't properly quote. This 'double count' is artificial. If interrogative sentences with interrogative pronouns are concept words, they can be quoted and can be recovered from a sentence. Concept words are not unsaturated in the sense Dummett claims they are: a predicate can occur on its own, namely when we ask a question. Concept words are expressions in their own right and not patterns that sentences can exemplify. There is therefore a single entity, the concept word, that does the two jobs for which Dummett needs simple and complex predicates.

10. To-do-list

I have tried to argue that concept words are unsaturated, while proper names are complete. A concept word contains the form of a sentence, its sense contains the form of a thought. There are further things to do:

The explanation of linguistic unsaturatedness via the model of the interrogative sentence does not fit all expressions Frege takes to be unsaturated. It fits only concept-words and distinguishes them from proper names. But we have covered the basic case, the atomic sentence and its constituents. It is reasonable to expect that we can use the ideas employed here to shed light on other forms of unsaturatedness. An interrogative sentence has argument places. If we understand the sense of the conjunctive or negator, we will take it similarly to be in need of completion. As someone who understands “Who is fat?” knows how to complete this concept word to assertoric sentences that can be used to answer the question, someone who can understand “It is not the case” knows how to complete the sentence operator to assertoric sentences whose content he wants to deny.

So far I have only been concerned with linguistic unsaturatedness. Does the interrogative view vindicate Frege’s thesis that concept words refer to something unsaturated? Lack of space prevents me from answering this question in detail. But consider this

Question meanings are functions that, when applied to the meaning of the answer, yield a proposition. (Krifka 2001, 2)

Perhaps one should phrase this better as:

Wh-Question meanings are functions that, when applied to the meaning of an expression that can take the place of the interrogative pronoun, yield a proposition.

Krifka takes the meaning of a question to be a function which, when completed by an argument, yields a proposition. There is, of course, something missing. When you answer a question you not only express a proposition, you have only answered my question if you have said something true. The concern for truth forces us to ascribe referents to expressions. Should we then not say:

Wh-Question referents are functions that, when applied to the referent of an expression that can take the place of the interrogative pronoun, yield a truth-value.

A final remark. If we ground our understanding of concept words in interrogative pronouns, we will be liberated from Quine's narrow understanding of variables:

Variables are pronouns, and make sense only in positions which are available for names. (Quine 1976, 198)

I have argued that variables are not interrogative pronouns, they are expressions that can complete the gap opened by an interrogative pronoun. Some, but certainly not all interrogative pronouns, can be replaced by names *salva congruitate* by names. 'What', 'how', 'when' can't. In his theory of quantification, Frege systematises this liberal understanding of quantification.

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